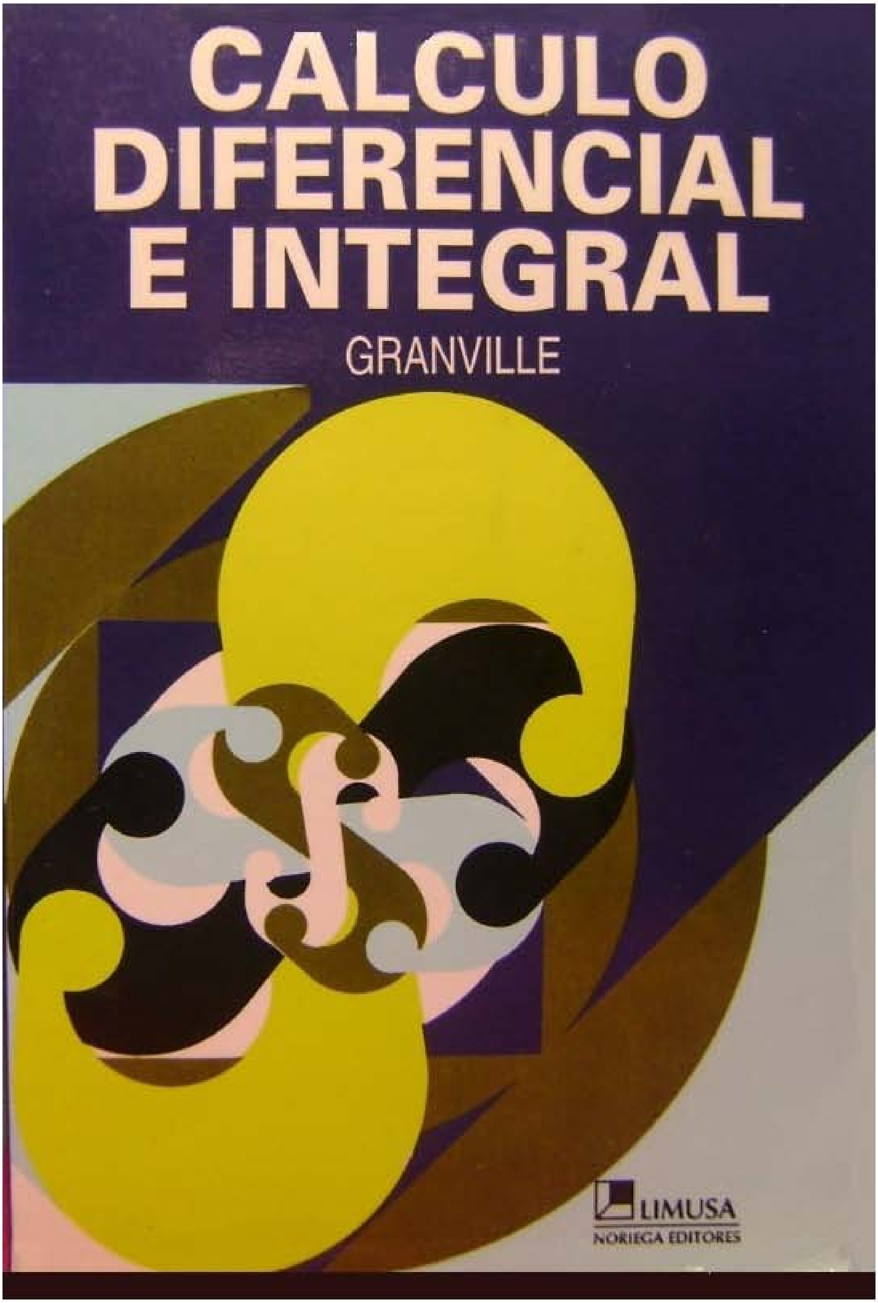
SOLUCIONARIO DE

CALCULO DIFERENCIAL E INTEGRAL - GRANVILLE



Problemas. Pagina 236

Verificar las siguientes Integraciones:

1. $\int x^4\, dx = x^5 + c$

$$\left.\begin{array}{l} v = x \\ dv = dx \\ n = 4 \end{array}\right\}$$ El diferencial esta completo, se procede a integrar.

$$\int x^4\, dx = \frac{x^{4+1}}{4+1} = \frac{x^5}{5} + c\,.$$

2. $\int \frac{dx}{x^2} =$

$$\int x^{-2}.dx$$

$$\left.\begin{array}{l} v = x \\ dv = dx \\ n = -2 \end{array}\right\}$$ El diferencial esta completo, se procede a integrar.

$$\int x^{-2}\, dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = -x^{-1} = -\frac{1}{x} + c\,.$$

3. $\int x^{2/3}\, dx$

$$\frac{x^{2/3+1}}{2/3+1} = \frac{x^{5/3}}{5/3} = \frac{3\,x^{5/3}}{5} + c\,.$$

4. $\int \frac{dx}{\sqrt{x}}$

$$\int x^{-1/2}.dx = \frac{x^{-1/2+1}}{-1/2+1} = \frac{x^{1/2}}{1/2} = 2x^{1/2} = 2\sqrt{x} + c\,.$$

5. $\int \frac{dx}{\sqrt[3]{x}} =$

$$\int \frac{dx}{x^{1/3}} = \int x^{-1/3}\, dx = \frac{x^{-1/3+1}}{-1/3+1} = \frac{x^{2/3}}{2/3} = \frac{3x^{2/3}}{2} + c \ .$$

6. $\int 3ay^2\, dy$

$$3a \int y^2\, dy = = 3a\left(\frac{y^{2+1}}{2+1}\right) = \frac{\cancel{3}\ ay^3}{\cancel{3}} = ay^3 + c \ .$$

7. $\int \frac{2\, dt}{t^2}$

$$2\int t^{-2}.\ dt = 2\left(\frac{t^{-2+1}}{-2+1}\right) = \frac{2t^{-1}}{-1} = -2.t^{-1} = -\frac{2}{t} + c \ .$$

8. $\int \sqrt{ax}\ .\ dx$

$\int (ax)^{1/2}.\ dx$ $\quad \left\{\begin{array}{l} v = ax \\ dv = a.dx \\ n = 1/2\ . \end{array}\right\}$ Falta (a) para completar, el diferencial.

$$\frac{1}{a}\int (ax)^{1/2}.\ a\ .dx = \frac{1}{a}\left(\frac{(ax)^{1/2+1}}{1/2+1}\right) = \frac{(ax)^{3/2}}{3/2(a)} = \frac{2(ax)^{3/2}}{3a} =$$

$$\frac{2(ax)^{2/2}(ax)^{1/2}}{3\ a} = \frac{2.\cancel{a}.x\ (ax)^{1/2}}{3\ \cancel{a}} = \frac{2\ x\ (ax)^{1/2}}{3} = \frac{2\ x\ \sqrt{ax}}{3} + c\ .$$

9. $\int \frac{dx}{\sqrt{2x}} =$

$$\int \frac{dx}{(2x)^{1/2}} = \int (2x)^{-1/2} =$$

$$\left.\begin{array}{l} v = 2x \\ dv = 2\,dx \\ n = -1/2 \end{array}\right\} \begin{array}{l} \text{Falta (2) para completar el diferencial.} \\ \text{Se aplica: } \int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c\,. \end{array}$$

$$\frac{1}{2}.\int (2x)^{-1/2}.2dx = \frac{1}{2}\left(\frac{(2x)^{-1/2+1}}{-1/2+1}\right) = \frac{(2x)^{1/2}}{2(1/2)} = \frac{(2x)^{1/2}}{2/2} = \frac{(2x)^{1/2}}{1} =$$

$$(2x)^{1/2} + c\,.$$

10. $\int \sqrt[3]{3t}\,.dt$

$$\int (3t)^{1/3}\,dt\,.$$

$$\left.\begin{array}{l} v = 3t \\ dv = 3\,dt \\ n = 1/3 \end{array}\right\} \begin{array}{l} \text{Falta (3) para completar el diferencial.} \\ \text{Se aplica: } \int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c\,. \end{array}$$

$$\frac{1}{3}\int (3t)^{1/3}.3dt = \frac{1}{3}\left(\frac{(3t)^{1/3+1}}{1/3+1}\right) = \frac{(3t)^{4/3}}{3(4/3)} = \frac{(3t)^{4/3}}{4} + c\,.$$

11. $\int (x^{3/2} - 2x^{2/3} + 5\sqrt{x} - 3)\,dx\,.$

$$\int x^{3/2}dx - 2\int x^{2/3}\,dx + 5\int \sqrt{x}\,dx - \int dx$$

$$\int x^{3/2}dx - 2\int x^{2/3}\,dx + 5\int (x)^{1/2}\,dx - \int dx$$

$$\frac{x^{3/2+1}}{3/2+1} - \frac{2\,x^{2/3+1}}{2/3+1} + \frac{5\,(x)^{1/2+1}}{1/2+1} - x + c\,.$$

$$\frac{x^{5/2}}{5/2} - \frac{2\,x^{5/3}}{5/3} + \frac{5\,(x)^{3/2}}{3/2} - x + c\,.$$

$$\frac{2x^{5/2}}{5} - \frac{6x^{5/3}}{5} + \frac{10(x)^{3/2}}{3} - x + c\,.$$

12. $\int \frac{4x^2 - 2\sqrt{x}}{x}\, dx$

$$\int \left(\frac{4x^2}{x} - \frac{2\sqrt{x}}{x} \right) dx = \int \left(4x - \frac{2x^{1/2}}{x^{2/2}} \right) dx =$$

$$\int (4x - 2x^{1/2} . x^{-2/2})\, dx = \int (4x - 2x^{-1/2})\, dx .$$

$$\int 4x\, dx - \int 2x^{-1/2}\, dx = 4\int x\, dx - 2\int x^{-1/2}\, dx .$$

$$4\left(\frac{x^{1+1}}{1+1} \right) - 2\left(\frac{x^{-1/2+1}}{-1/2+1} \right) = \frac{4 . x^2}{2} - \frac{2 . x^{1/2}}{1/2} = 2x^2 - 4x^{1/2} =$$

$$2x^2 - 4\sqrt{x} + c .$$

13. $\int \left(\frac{x^2}{2} - \frac{2}{x^2} \right) dx .$

$$\int \frac{x^2}{2}\, dx - \int \frac{2}{x^2}\, dx = \frac{1}{2}\int x^2\, dx - 2\int x^{-2}\, dx =$$

$$\frac{1}{2}\left(\frac{x^{2+1}}{2+1} \right) - 2\left(\frac{x^{-2+1}}{-2+1} \right) = \frac{x^3}{2(3)} - \frac{2.x^{-1}}{-1} = \frac{x^3}{6} + \frac{2}{x} + c .$$

14. $\int \sqrt{x}(3x - 2)\, dx$

$$\int (3x. \sqrt{x} - 2. \sqrt{x})\, dx = \int (3x.x^{1/2} - 2x^{1/2})\, dx = \int (3x^{3/2} - 2x^{1/2})\, dx .$$

$$\int 3x^{3/2}\, dx - \int 2x^{1/2}\, dx = 3\int x^{3/2}\, dx - 2\int x^{1/2}\, dx =$$

$$3\left(\frac{x^{3/2+1}}{3/2+1} \right) - 2\left(\frac{x^{1/2+1}}{1/2+1} \right) = \left(3\,\frac{x^{3/2+1}}{3/2+1} - 2\,\frac{x^{1/2+1}}{1/2+1} = \right)$$

$$\frac{3x^{5/2}}{5/2} - \frac{2x^{3/2}}{3/2} = \frac{6x^{5/2}}{5} - \frac{4x^{3/2}}{3} + c .$$

15. $\int \frac{x^3 - 6x + 5}{x}\, dx = \frac{x^3}{3} - 6x + 5 \ln x + c$.

$\int \left(\frac{x^3}{x} - \frac{6x}{x} + \frac{5}{x} \right) dx = \int \left(x^2 - 6 + \frac{5}{x} \right) dx = \int x^2\, dx - 6 \int dx + 5 \int \frac{dx}{x}$

$\frac{x^{2+1}}{2+1} - 6(x) + 5(\ln x) = \frac{x^3}{3} - 6x + 5 \ln x + c$.

16. $\int \sqrt{a + bx}\, dx = \frac{2(a + bx)^{3/2}}{3b} + c$.

$\int (a + bx)^{1/2}\, dx$.

$\left.\begin{array}{l} v = (a + bx) \\ dv = b\, dx \\ n = 1/2 \end{array}\right\}$ Falta (b) para completar el diferencial. $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$.

$\frac{1}{b} \int (a + bx)^{1/2}\, b dx = \frac{1}{b} \left(\frac{(a + bx)^{1/2+1}}{1/2+1} \right) = \frac{(a + bx)^{3/2}}{b(3/2)} = \frac{(a + bx)^{3/2}}{\frac{3b}{2}} =$

$\frac{2(a + bx)^{3/2}}{3b} + c$

17. $\int \frac{dy}{\sqrt{a - by}}$.

$\int \frac{dy}{(a - by)^{1/2}} = \int (a - by)^{-1/2}\, dy =$

$\left.\begin{array}{l} v = (a - by) \\ dv = - b\, dy \\ n = - 1/2 \end{array}\right\}$ Falta (-b) para completar el diferencial. $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$

$$\frac{-1}{b}\int (a - by)^{-1/2}.(- b)\, dy$$

$$\frac{-1}{b}\left(\frac{(a - by)^{-1/2+1}}{-1/2+1}\right) = -\frac{(a - by)^{1/2}}{b(1/2)} = -\frac{(a - by)^{1/2}}{b/2} = \frac{-2\,(a - by)^{1/2}}{b} + c.$$

18. $\int (a + bt)^2\, dt = \dfrac{(a + bt)^3}{3} + c\ .$

$$\left.\begin{array}{l} v = (a + bt) \\ dv = b\, dt \\ n = 2 \end{array}\right\} \begin{array}{l} \text{Falta (b), para completar el diferencial, se aplica:} \\ \int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c\ . \end{array}$$

$$\frac{1}{b}\left(\int (a + bt)^2.b\, dt\right) = \frac{(a + bt)^{2+1}}{b(2+1)} = \frac{(a + bt)^3}{3b} + c\ .$$

19. $\int x\,(2 + x^2)^2\, dx = \dfrac{(2 + x^2)^3}{6}.$

$$\int (2 + x^2)^2.\ x\, dx$$

$$\left.\begin{array}{l} v = (2 + x^2) \\ dv = 2x\, dx \\ n = 2 \end{array}\right\} \begin{array}{l} \text{Falta (2), se aplica:} \int v^n = v^{n+1}/n+1 + c\ . \\ \dfrac{1}{2}\int (2 + x^2)^2.\ 2x\, dx = \dfrac{1}{2}\left(\dfrac{(2 + x^2)^{2+1}}{2+1}\right) = \dfrac{(2 + x^2)^3}{2(3)} = \dfrac{(2 + x^2)^3}{6} + c \end{array}$$

20. $\int y\,(a - by^2)\, dy = -\dfrac{(a - by^2)^2}{4b} + c\ .$

$$\int (a - by^2)\ .\ y\, dy\ .$$

$$\left.\begin{array}{l} v = (a - by^2) \\ dv = -2by\, dy \\ n = 1 \end{array}\right\} \begin{array}{l} \text{Falta (-2b),para completar el diferencial.} \\ \text{Se aplica:} \int v^n = v^{n+1}/n+1 + c\ . \end{array}$$

$$\int (a - by^2)\ .\ y\, dy = \frac{-1}{2b}\left(\frac{(a - by^2)^{1+1}}{1+1}\right) = -\frac{(a - by)^2}{2b(2)} = -\frac{(a - by^2)}{4b} + c.$$

21. $\int t\sqrt{2t^2+3}\,dt = \dfrac{(2t^2+3)^{3/2}}{6} + c$.

$\int (2t^2+3)^{1/2}.\ t\,dt$

$\left.\begin{array}{l} v = (2t^2+3) \\ dv = 4t\,dt\ . \\ n = 1/2 \end{array}\right\}$ Falta (4) para completar el diferencial. Se aplica: $\int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c$.

$\dfrac{1}{4}\int (2t^2+3)^{1/2}.\ 4t\,dt = \dfrac{1}{4}\left(\dfrac{(2t^2+3)^{1/2+1}}{1/2+1}\right) = \dfrac{(2t^2+3)^{3/2}}{4(3/2)} = \dfrac{(2t^2+3)^{3/2}}{12/2} =$

$\dfrac{(2t^2+3)^{1/2}}{6} + c$.

22. $\int x(2x+1)^2\,dx = x^4 + \dfrac{4x^3}{3} + \dfrac{x^2}{2} + c$.

Primero solucionamos el producto notable:

$(2x+1)^2 = 4x^2 + 4x + 1$.

$\int x(4x^2+4x+1) = \int (4x^3+4x^2+x)\,dx$.

$\int 4x^3\,dx + \int 4x^2\,dx + \int x\,dx = 4\int x^3\,dx + 4\int x^2\,dx + \int x\,dx$.

$4\left(\dfrac{x^{3+1}}{3+1}\right) + 4\left(\dfrac{x^{2+1}}{2+1}\right) + \dfrac{x^{1+1}}{1+1} = \dfrac{4x^4}{4} + \dfrac{4x^3}{3} + \dfrac{x^2}{2} =$

$x^4 + \dfrac{4x^3}{3} + \dfrac{x^2}{2} + c$.

23. $\int \dfrac{4x^2\,dx}{\sqrt{x^3+8}}$

$\int (x^3+8)^{-1/2} \,.\, 4x^2\,dx$

$$\left.\begin{array}{l} v = (x^3+8) \\ dv = 3x^2\,dx \\ n = -1/2 \end{array}\right\}$$

Falta (3) para completar el diferencial.
Se aplica: $\int v^n\,dv = \frac{v^{n+1}}{n+1} + c$.

El # 4 sale fuera de la integral porque no nos va a servir en dv.

$$\frac{4}{3}\int (x^3+8)^{-1/2} \,.\, 3x^2\,dx = \frac{4}{3}\left(\frac{(x^3+8)^{-1/2+1}}{-1/2+1}\right) = \frac{4(x^3+8)^{1/2}}{3(1/2)} =$$

$$\frac{4(x^3+8)^{1/2}}{3/2} = \frac{2\{4(x^3+8)^{1/2}\}}{3} = \frac{8(x^3+8)^{1/2}}{3} = \frac{8\sqrt{(x^3+8)}}{3} + c \,.$$

24. $\int \frac{6z\,dz}{(5-3z^2)^2}$.

$\int (5-3z^2)^{-2}.6z\,dz$

$$\left.\begin{array}{l} v = (5-3z^2) \\ dv = -6z \\ n = -2 \end{array}\right\}$$

A la integral original para que se integre solo le falta el signo negativo.

$-\int (5-3z^2)^{-2}.\,(-)\,6z\,dz$

$$-\frac{(5-3z^2)^{-2+1}}{-2+1} = -\frac{(5-3z^2)^{-1}}{-1} = (5-3z^2)^{-1} = \frac{1}{(5-3z^2)} + c \,.$$

25. $\int (\sqrt{a}-\sqrt{x})^2\,dx$ **.**

Solucionando el producto notable: $(\sqrt{a}-\sqrt{x})^2 = a - 2\sqrt{a}.\sqrt{x} + x$.

$\int \{(\sqrt{a})^2 - 2\sqrt{a}.\sqrt{x} + (\sqrt{x})^2\}\,dx = \int (a - 2\sqrt{a}.\sqrt{x} + x)\,dx$.

$\int a\,dx - \int 2\sqrt{a}.\sqrt{x} + \int x\,dx = a\int dx - 2\sqrt{a}\int \sqrt{x}\,dx + \int x\,dx$.

$$a\int dx - 2a^{1/2}\int x^{1/2}\,dx + \int x\,dx = a.\,x - \frac{2a^{1/2}.x^{1/2+1}}{1/2+1} + \frac{x^{1+1}}{1+1} =$$

$$ax - \frac{2a^{1/2}x^{3/2}}{3/2} + \frac{x^2}{2} = ax - \frac{4\,x^{2/2}\,a^{1/2}\,x^{1/2}}{3} + \frac{x^2}{2} =$$

$$ax - \frac{4x\sqrt{a}.\sqrt{x}}{3} + \frac{x^2}{2} = ax - \frac{4x\sqrt{ax}}{3} + \frac{x^2}{2} + c\ .$$

26. $\displaystyle\int \frac{(\sqrt{a} - \sqrt{x})^2}{\sqrt{x}}\,dx$

$$\left.\begin{array}{l} v = (\sqrt{a} - \sqrt{x}) \\ dv = \dfrac{-1}{2\sqrt{x}}dx\ . \\ n = 2 \end{array}\right\} \begin{array}{l} \text{Falta (-1/2) para completar el diferencial.} \\ \text{Se aplica: } \displaystyle\int v^n\,dv = \frac{v^{n+1}}{n+1} + c\ . \end{array}$$

$$\int (\sqrt{a} - \sqrt{x})^2.\frac{1}{\sqrt{x}}.dx = -2\int (\sqrt{a} - \sqrt{x})^2\left(-\frac{1}{2\sqrt{x}}\right)dx$$

$$-2\left(\frac{(\sqrt{a} - \sqrt{x})^{2+1}}{2+1}\right) = \frac{-2(\sqrt{a} - \sqrt{x})^3}{3} + c\ .$$

27. $\displaystyle\int \sqrt{x}\left(\sqrt{a} - \sqrt{x}\right)^2.dx$

$$\int \sqrt{x}\{(\sqrt{a})^2 - 2\sqrt{a}.\sqrt{x} + (\sqrt{x})^2\}\,dx = \int \sqrt{x}(a - 2\sqrt{a}.\sqrt{x} + x)\,dx$$

$$\int (a\sqrt{x} - 2\sqrt{a}.\sqrt{x}.\sqrt{x} + x.\sqrt{x})dx = \int \{ax^{1/2} - 2^{a1/2}.(\sqrt{x})^2 + x^{2/2}.x^{1/2}\}dx$$

$$\int \{ax^{1/2} - 2a^{1/2}\,x + x^{3/2}\}\,dx = a\int x^{1/2}\,dx - 2a^{1/2}\int x\,dx + \int x^{3/2}\,dx =$$

$$a\left(\frac{x^{1/2+1}}{1/2+1}\right) - 2a^{1/2}\left(\frac{x^{1+1}}{1+1}\right) + \frac{x^{3/2+1}}{3/2+1} = \frac{a.x^{3/2}}{3/2} - \frac{2a^{1/2}.x^2}{2} + \frac{x^{5/2}}{5/2} =$$

$$\frac{2a\,.x^{3/2}}{3} - a^{1/2}.x^2 + \frac{2x^{5/2}}{5} = \frac{2ax^{3/2}}{3} - x^2\sqrt{a} + \frac{2x^{5/2}}{5} + c\ .$$

28. $\displaystyle\int \frac{t^3\,dt}{\sqrt{a^4 + t^4}}\ .$

$\displaystyle\int (a^4 + t^4)^{-1/2}.t^3\,dt\ .$ $\quad\left\{ v = (a^4 + t^4) \right\}$ Falta (4)para completar el

$dv = 4t^3\ dt$ diferencial, se aplica:
$n = -1/2$ $\int v^n\ dv = v^{n+1}/n+1 + c$.

$$\frac{1}{4}\int (a^4+t^4)^{-1/2}.(4)t^3\ dt = \frac{1}{4}\left\{\frac{(a^4+t^4)^{-1/2+1}}{-1/2+1}\right\} = \frac{(a^4+t^4)^{1/2}}{4(1/2)} =$$

$$\frac{(a^4+t^4)^{1/2}}{4/2} = \frac{2(a^4+t^4)^{1/2}}{4} = \frac{(a^4+t^4)^{1/2}}{2} = \sqrt{(a^4+t^4)} + c\ .$$

29. $\displaystyle\int \frac{dy}{(a+by)^3}$.

$\int (a+by)^{-3}\ dy$

$\left.\begin{array}{l} v = (a+by) \\ dv = b\ dy \\ n = -3 \end{array}\right\}$ Falta (b) para completar el diferencial. Se aplica: Se aplica: $\int v^n\ dv = \dfrac{v^{n+1}}{n+1} + c$.

$$\frac{1}{b}\int (a+by)^{-3}.(b)dy$$

$$\frac{1}{b}\left(\frac{(a+by)^{-3+1}}{-3+1}\right) = \frac{(a+by)^{-2}}{b(-2)} = \frac{(a+by)^{-2}}{-2b} = -\frac{1}{2b(a+by)^2} + c\ .$$

30. $\displaystyle\int \frac{x\ dx}{(a+bx^2)^3}$.

$\int (a+bx^2)^{-3}.x\ dx$

$\left.\begin{array}{l} v = (a+bx^2) \\ dv = 2bx.dx \end{array}\right\}$ Falta (2b) para completar el diferencial. Se aplica: Se aplica: $\int v^n\ dv = \dfrac{v^{n+1}}{n+1} + c$.

$$\frac{1}{2b}\int (a+bx^2)^{-3}.(2b)x\ dx$$

$$\frac{1}{2b}\left(\frac{(a+bx^2)^{-3+1}}{-3+1}\right)=\frac{(a+bx^2)^{-2}}{(2b)(-2)}=-\frac{1}{4b(a+bx^2)^2}+c.$$

31. $\int \frac{t^2\,dt}{(a+bt^3)^2}$.

$$\int (a+bt^3)^2.t^2\,dt$$

$$\left.\begin{array}{l} v=(a+bt^3) \\ dv=3bt^2\,dt \\ n=2 \end{array}\right\} \begin{array}{l}\text{Falta (3b) para completar el diferencial.} \\ \text{Se aplica: } \int v^n\,dv=\frac{v^{n+1}}{n+1}+c.\end{array}$$

$$\frac{1}{3b}\int (a+bt^3)^{-2}.(3b)t^2\,dt=\frac{(a+bt^3)^{-2+1}}{3b(-2+1)}=\frac{(a+bt^3)^{-1}}{3b(-1)}=$$

$$\frac{(a+bt^3)^{-1}}{-3b}=-\frac{1}{3b(a+bt^3)}+c.$$

32. $\int z(a+bz^3)^2\,dz$.

Desarrollando el producto notable: $(a+bz^3)^2$, obtenemos ,

$$\int z\,(a^2+2abz^3+b^2z^6)\,dz$$

$$\int (a^2z+2abz^4+b^2z^7)\,dz$$

$$a^2\int z\,dz+2ab\int z^4\,dz+b^2\int z^7\,dz$$

$$a^2\left(\frac{z^{1+1}}{1+1}\right)+2ab\left(\frac{z^{4+1}}{4+1}\right)+b^2\left(\frac{z^{7+1}}{7+1}\right)=\frac{a^2z^2}{2}+\frac{2abz^5}{5}+\frac{b^2z^8}{8}+c.$$

33. $\int x^{n-1}\sqrt{a+bx^n}\,dx$

$$\int (a+bx^n)^{1/2}.\,x^{n-1}\,dx$$

$$\left.\begin{array}{l} v = (a + bx^n) \\ dv = nbx^{n-1}\ dx \\ n = 1/2 \end{array}\right\} \begin{array}{l} \text{Falta (nb) para completar el diferencial.} \\ \text{Se aplica: } \int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c\ . \end{array}$$

$$\frac{1}{nb} \int (a + bx^n)^{1/2}.\ (nb)\ x^{n-1}\ dx$$

$$\frac{(a + bx^n)^{1/2+1}}{1/2+1} = \frac{(a + bx^n)^{3/2}}{3/2} = \frac{2(a + bx^n)^{3/2}}{3} + c\ .$$

34. $\displaystyle \int \frac{(2x + 3)\ dx}{\sqrt{x^2 + 3x}}$

$$\int (x^2 + 3x)^{-1/2}.\ (2x + 3)\ dx$$

$$\left.\begin{array}{l} v = (x^2 + 3x) \\ dv = 2x + 3 \\ n = -1/2 \end{array}\right\} \begin{array}{l} \text{El diferencial esta completo, se procede a integrar.} \\ \text{Se aplica: } \int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c\ . \end{array}$$

$$\int (x^2 + 3x)^{-1/2}.\ (2x + 3)\ dx$$

$$\frac{(x^2 + 3x)^{-1/2+1}}{-1/2 + 1} = \frac{(x^2 + 3x)^{1/2}}{1/2} = 2(x^2 + 3x)^{1/2} = 2\ \sqrt{x^2 + 3x}\ +\ c\ .$$

35. $\displaystyle \int \frac{(x^2 + 1)\ dx}{\sqrt{x^3 + 3x}}$.

$$\int (x^3 + 3x)^{-1/2}.\ (x^2 + 1)\ dx$$

$$\left.\begin{array}{l} v = (x^3 + 3x) \\ dv = 3x^2 + 3\ dx = 3(x^2 + 1)\ dx \\ n = -1/2 \end{array}\right\} \begin{array}{l} \text{Falta (3) para completar el} \\ \text{diferencial.} \end{array}$$

$$\frac{1}{3} \int (x^3 + 3x)^{-1/2}.(3)(x^2 + 1)\ dx = \frac{(x^3 + 3x)^{-1/2+1}}{3(-1/2+1)} = \frac{(x^3 + 3x)^{1/2}}{3(1/2)} =$$

$$\frac{(x^3+3x)^{1/2}}{3/2} = \frac{2(x^3+3x)^{1/2}}{3} = \frac{2\sqrt{(x^3+3x)}}{3} + c .$$

36. $\int \frac{(2+\ln x)\,dx}{x}$

$$\int (2+\ln x).\ \frac{1}{x}\ dx$$

$$\left.\begin{array}{l} v = (2+\ln x) \\ dv = \frac{1}{x}\ dx \\ n = 1 \end{array}\right\}$$ Falta 1/x para completar el diferencial. Se aplica: $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$.

$$\int (2+\ln x).\ \frac{1}{x}\ dx = \frac{(2+\ln x)^{1+1}}{1+1} = \frac{(2+\ln x)^2}{2} + c .$$

37. $\int \mathbf{sen^2x \cos x\, dx}$

$\int (\text{senx})^2 .\ \cos x\ dx .$ $\quad \left\{\begin{array}{l} v = (\text{senx}) \\ dv = \cos x\ dx \\ n = 2 \end{array}\right\}$ El diferencial esta completo,se procede a integrar.

$$\int (\text{senx})^2 \cos x\ dx = \frac{(\text{senx})^{2+1}}{2+1} = \frac{(\text{senx})^3}{3} + c .$$

38. $\int \mathbf{sen\ ax \cos ax\, dx}$

$$\left.\begin{array}{l} v = \text{sen ax} \\ dv = (\cos ax)(a)\ dx = a \cos ax\ dx \\ n = 1 \end{array}\right\}$$ Falta (a) para completar el diferencial.Se aplica: $\int v^n\, dv = \frac{vn^{+1}}{n+1} + c$.

$$\frac{1}{a}\int (\text{sen ax}) . (a)\cos ax\ dx = \frac{(\text{sen ax})^{1+1}}{a(1+1)} = \frac{(\text{sen ax})^2}{2a} = \frac{\text{sen}^2 ax}{2a} + c .$$

39. $\int \sin 2x \cos^2 2x\, dx$

$\int (\cos 2x)^2 .\ \text{sen } 2x\, dx$

$v = (\cos 2x)$
$dv = (-\text{sen } 2x)(2)\, dx = -2\text{sen } 2x$
$n = 2$

Falta (-2) para completar el diferencial
Se aplica: $\int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c$.

$\dfrac{-1}{2}\int (\cos 2x)^2.(-2)\text{sen } 2x\, dx = -\dfrac{(\cos 2x)^{2+1}}{2(2+1)} = -\dfrac{(\cos 2x)^3}{2(3)} =$

$-\dfrac{\cos^3 2x}{6} + c$.

40. $\int \text{tg } \dfrac{x}{2} \sec^2 \dfrac{x}{2} dx$

$v = \text{tg } x/2$
$dv = \dfrac{1}{2} \sec^2 \dfrac{x}{2}$
$n = 1$

falta (1/2) para completar el diferencial.

$2\int \text{tg } \dfrac{x}{2}\left(\dfrac{1}{2}\right) . \sec^2 \dfrac{x}{2} dx = \dfrac{\dfrac{2\left[\text{tg } \frac{x}{2}\right]^{1+1}}{2}}{1+1} = \dfrac{\dfrac{\cancel{2}\left[\text{tg } \frac{x}{2}\right]^2}{2}}{\cancel{2}} =$

$\text{tg}^2\left(\dfrac{x}{2}\right) = \left[\text{tg}^2 \dfrac{x}{2}\right] + c$.

41. $\int \dfrac{\cos ax\, dx}{\sqrt{b + \text{sen } ax}}$.

$\int (b + \text{sen } ax)^{-1/2} . \cos ax\, dx$.

$v = (b + \text{sen } ax)$

Falta (a) para completar el

$dv = \cos ax . a\,dx = a \cos ax\,dx$ diferencial: Se aplica:

$n = -1/2$ $\qquad \int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c$.

$$\frac{1}{a}\int (b + \operatorname{sen} ax)^{-1/2} .(a) \cos ax\,dx = \frac{(b + \operatorname{sen} ax)^{-1/2+1}}{a(-1/2+1)} =$$

$$\frac{(b + \operatorname{sen} ax)^{1/2}}{a(1/2)} = \frac{(b + \operatorname{sen} ax)^{1/2}}{a/2} = \frac{2(b + \operatorname{sen} ax)^{1/2}}{a} =$$

$$\frac{2\sqrt{b + \operatorname{sen} ax}}{a} + c .$$

42. $\displaystyle\int \left(\frac{\sec x}{1 + \operatorname{tg} x}\right)^2 dx$

$$\int \frac{\sec^2 x}{(1 + \operatorname{tg}^2 x)}\,dx$$

$$\int (1 + \operatorname{tg} x)^{-2}. \operatorname{Sec}^2 x\,dx .$$

$v = (1 + \operatorname{tg} x)$
$dv = \sec^2 x\,dx$
$n = -2$

El diferencial esta completo, se procede a integrar.

$$\frac{(1 + \operatorname{tg} x)^{-2+1}}{-2+1} = \frac{(1 + \operatorname{tg} x)^{-1}}{-1} = -\frac{1}{(1 + \operatorname{tg} x)} + c .$$

43. $\displaystyle\int \frac{dx}{2 + 3x}$.

$v = 2 + 3x$
$dv = 3\,dx$

Falta (3) para completar el diferencial.
Se aplica: $\displaystyle\int \frac{dv}{v} = \ln v + c$.

$$\frac{1}{3}\int \frac{(3)\ dx}{2 + 3x} = \frac{1}{3} \ln (2 + 3x) + c .$$

44. $\int \dfrac{x^2\,dx}{2+x^3}$.

$v = 2 + x^3$
$dv = 3x^2\,dx$
Falta (3) para completar el diferencial.
Se aplica: $\int \dfrac{dv}{v} = \ln v + c$.

$$\frac{1}{3}\int \frac{(3)\,x^2\,dx}{2+x^3} = \frac{1}{3}\ln(2+x^3) = \frac{\ln(2+x^3)}{3} + c \; .$$

45. $\int \dfrac{t\;dt}{a+bt^2}$.

$v = a + bt^2$
$dv = 2bt$
Falta (2b) para completar el diferencial.
Se aplica : $\int \dfrac{dv}{v} = \ln v + c$.

$$\frac{1}{2b}\int \frac{(2b)\,t\,dt}{(a+bt^2)} = \frac{1}{2b}.\ln(a+bt^2) = \frac{\ln(a+bt^2)}{2b} + c \; .$$

46. $\int \dfrac{(2x+3)\,dx}{x^2+3x}$

$v = x^2 + x$
$dv = (2x + 3)$
El diferencial esta completo, se procede a integrar.

$$\int \frac{(2x+3)\,dx}{x^2+3x} = \ln(x^2+3x) + c \; .$$

47. $\int \dfrac{(y+2)\,dy}{y^2+4y}$

$v = y^2 + 4y$
$dv = 2y + 4\,dy = 2(y+2)\,dy$
Falta (2) para completar el diferencial .Se aplica:
$\int \dfrac{dv}{v} = \ln v + c$.

$$\frac{1}{2}\int\frac{(2)(y+2)\,dy}{(y^2+4y)} = \frac{1}{2}.\ln(y^2+4y) = \frac{\ln(y^2+4y)}{2} + c\,.$$

48. $\displaystyle\int\frac{e^{\theta}\,d\theta}{a+be^{\theta}}$.

$$\left.\begin{array}{l} v = a+be^{\theta} \\ dv = be^{\theta}\,d\theta \end{array}\right\}\begin{array}{l}\text{Falta (b) para completar el diferencial.}\\ \text{Se aplica: } \int dv/v = \ln v + c\,.\end{array}$$

$$\frac{1}{b}\left(\int\frac{e^{\theta}(b)\,d\theta}{a+be^{\theta}}\right).$$

$$\frac{\ln(a+be^{\theta})}{b} + c$$

49. $\displaystyle\int\frac{\text{sen } x\,dx}{1-\cos x}$.

$$\left.\begin{array}{l} v = 1-\cos x \\ dv = -(-\text{sen } x)\,dx = \text{sen } x\,dx\,. \end{array}\right\}\begin{array}{l}\text{El diferencial esta completo.}\\ \text{Se procede a integrar.}\end{array}$$

□ $\ln(1-\cos x) + c$.

50. $\displaystyle\int\frac{\sec^2 y\,dy}{a+b\,\text{tg } y}$.

$$\left\{\begin{array}{l} v = a+b\,\text{tg } y\ .\ \text{Falta (b), para completar el diferencial} \\ dv = b\sec^2 y\,dy \end{array}\right\}$$

$$\frac{1}{b}\left(\int\frac{(b)\sec^2 y\,dy}{a+b\,\text{tg } y}\right) = \frac{1}{b}.\ \ln(a+b\,\text{tg } y) = \frac{\ln(a+b\,\text{tg } y)}{b} + c\,.$$

51. $\displaystyle\int\frac{(2x+3)\,dx}{x+2}$

Efectuamos la división:
$$\begin{array}{r|l} 2x + 3 & x + 2 \\ \underline{-2x - 4} & 2 \\ -1 & \end{array}$$

El resultado es:

$2 + \dfrac{-1}{x+2} = 2 - \dfrac{1}{x+2}$. Sustituyendo en la integral .

$$\int \left[2 - \frac{1}{x+2} \right] dx = 2\int dx - \int \frac{dx}{x+2} = 2x - \ln(x+2) + c$$

52. $\displaystyle \int \frac{x^2 + 2}{x+1}\, dx$

Efectuamos la división:
$$\begin{array}{r|l} \cancel{x^2} \quad + 2 & x + 1 \\ \underline{-\cancel{x^2} - x} & x - 1 \\ -\cancel{x} & \\ \underline{+\cancel{x} + 2} & \\ +2 & \end{array}$$

El resultado es:

$(x - 1) + \dfrac{3}{x+1}$. Sustituyendo en la Integral.

$$\int \left[x - 1 + \frac{3}{x+1} \right] dx$$

$$\int x\, dx - \int dx + 3\int \frac{dx}{x+1} .$$

$$\frac{x^{1+1}}{1+1} - x + 3\ln(x+1) = \frac{x^2}{2} - x + 3\ln(x+1) + c .$$

53. $\int \frac{(x+4)\,dx}{2x+3}$

Efectuamos la división:

$$\begin{array}{r|l} \cancel{x} + 4 & 2x+3 \\ \hline -\cancel{x} - 3/2 & 1/2 \\ \hline -x + 5/2 & \end{array}$$

El resultado es: $\frac{1}{2} + \frac{\frac{5}{2}}{2x+3}$. Sustituyendo en la Integral.

$$\int \left(\frac{1}{2} + \frac{5/2}{2x+3}\right) dx$$

$$\int \frac{1}{2}\,dx + \frac{5}{2}\cdot\frac{1}{2}\int \frac{(2)dx}{2x+3} \cdot \left\{\begin{array}{l} v = 2x+3 \\ dv = 2\,dx \end{array}\right.$$

$$\frac{1}{2}\int dx + \frac{5}{4}\int \frac{(2)\,dx}{2x+3} = \frac{1}{2}\,x + \frac{5}{4}\ln(2x+3) =$$

$$\frac{x}{2} + \frac{5\ln(2x+3)}{4} + c\ .$$

54. $\int \frac{e^{2s}\,ds}{e^{2s}+1}$.

$\left\{\begin{array}{l} v = e^{2s}+1 \\ dv = 2e^{2s}. \end{array}\right\}$ El diferencial esta incompleto, falta (2) y se le opone 1/2.

$$\frac{1}{2}\int \frac{(2)e^{2s}\,ds}{e^{2s}+1} = \frac{1}{2}.\ \ln(e^{2s}+1) = \frac{\ln(e^{2s}+1)}{2} + c\ .$$

55. $\int \left(\frac{ae^{\theta}+b}{ae^{\theta}-b}\right) d\theta$

Efectuamos la división:

$$\begin{array}{r|l} b + ae^{\theta} & \underline{-b + ae^{\theta}} \\ \underline{-b + ae^{\theta}} & -1 \\ \cancel{+}2ae^{\theta} & \end{array}$$

El resultado es:

$$\left(-1 + \frac{2ae^{\theta}}{-b + ae^{\theta}} \right)$$

Para la 2^{da} integral:

$$\left.\begin{array}{l} v = -b + ae^{\theta} \\ dv = ae^{\theta} d\theta \end{array}\right\}$$

$$\int -1 + \frac{2\,ae^{\theta}}{-b + ae^{\theta}} \quad d\theta = -\int d\theta + 2\int \frac{ae^{\theta}\, d\theta}{-b + ae^{\theta}} =$$

$$-\theta + 2\ln(-b + ae^{\theta}) = 2\ln(ae^{\theta} - b) - \theta + c\,.$$

56. $\displaystyle \int \frac{2x\,dx}{\sqrt[3]{(6-5x^2)}}\,.$

$$\int (6 - 5x^2)^{-1/3}.2x\,dx$$

$$\left\{\begin{array}{l} v = (6 - 5x^2) \\ dv = -10x\,dx \\ n = -1/3\,. \end{array}\right\}$$ El diferencial esta incompleto, falta (- 5) .

$$-\frac{1}{5}\int \underline{(6 - 5x^2)^{-1/3}(-5)2x\,dx} = -\frac{1}{5}.\frac{(6 - 5x^2)^{-1/3+1}}{-1/3+1} = \frac{-(6 - 5x^2)^{2/3}}{5(2/3)} =$$

$$-\frac{3(6 - 5x^2)^{2/3}}{10} + c.$$

57. $\displaystyle \int (x^3 + 3x^2)\,dx$

$$\int x^3\,dx + 3\int x^2\,dx$$

$$\frac{x^{3+1}}{3+1} + \frac{3.x^{2+1}}{2+1} = \frac{x^4}{4} + \frac{3x^3}{3} = \frac{x^4}{4} + x^3 = c\,.$$

58. $\int \frac{x^2 - 4}{x^4} . dx$

Desarrollando: $\frac{x^2 - 4}{x^4} = \frac{x^2}{x^4} - \frac{4}{x^4} = \frac{1}{x^2} - \frac{4}{x^4}$.

Sustituyendo en la integral .

$$\int \left[\frac{1}{x^2} - \frac{4}{x^4} \right] dx = \int \frac{1}{x^2} dx - 4 \int \frac{dx}{x^4} = \int x^{-2} dx - 4\int x^{-4} dx$$

$$\frac{x^{-2+1}}{-2+1} - \frac{4.x^{-4+1}}{-4+1} = \frac{x^{-1}}{-1} - \frac{4x^{-3}}{-3} = -\frac{1}{x} + \frac{4}{3x^3} + c .$$

59. $\int \left(\frac{\sqrt{5x}}{5} + \frac{5}{\sqrt{5x}} \right) dx$

$$\frac{1}{5} \int \sqrt{5x}\, dx + 5 \int \frac{dx}{\sqrt{5x}} = \frac{1}{5} \int (5x)^{1/2} dx + 5 \int (5x)^{-1/2} dx.$$

$$\left\{ \begin{array}{l} v = 5x \\ dv = 5\ dx \\ n = 1/2 \end{array} \right\} \qquad \left\{ \begin{array}{l} v = 5x \\ dv = 5\ dx \\ n = -1/2 \end{array} \right\}$$ Completando el diferencial a ambas integrales.

$$\frac{1}{5} . \frac{1}{(5)} \int (5x)^{1/2}.(5)dx + 5.\frac{1}{5} \int (5x)^{-1/2} (5)dx =$$

$$\frac{1}{25} . \frac{(5x)^{1/2+1}}{1/2 + 1} + \frac{(5x)^{-1/2+1}}{-1/2+1} =$$

$$\frac{(5x)^{3/2}}{25(3/2)} + \frac{(5x)^{-1/2+1}}{1/2} = \frac{2(5x)^{3/2}}{5(5)(3)} + \frac{2(5x)^{1/2}}{1} =$$

$$\frac{2(\cancel{5}x)(5x)^{1/2}}{\cancel{5}(5)(3)} + 2(5x)^{1/2} = \frac{2x(5x)^{1/2}}{15} + 2(5x)^{1/2} =$$

$$2(5x)^{1/2} \left\{ \frac{x}{15} + 1 \right\} = 2\sqrt{5.x} \left\{ \frac{x + 15}{15} \right\} + c .$$

60. $\int \sqrt[3]{by^2}$

$$\int \sqrt[3]{b}.\sqrt[3]{y^2}.dy = \sqrt[3]{b}\int \sqrt[3]{y^2}.dy = \sqrt[3]{b}\int y^{2/3}.dy = \sqrt[3]{b}\left(\frac{y^{2/3+1}}{2/3+1}\right) =$$

$$\sqrt[3]{b}\left(\frac{y^{2/3+1}}{2/3+1}\right) = b^{1/3}\left(\frac{y^{5/3}}{5/3}\right) = \frac{3b^{1/3}y^{5/3}}{5} = \frac{3\sqrt[3]{by^5}}{5} + c.$$

61. $\int \frac{dt}{t\sqrt{2t}}$

$$\int \frac{dt}{t.t^{1/2}.2^{1/2}} = \frac{1}{2^{1/2}}\int \frac{dt}{t^{1+1/2}} = \frac{1}{\sqrt{2}}.\int \frac{dt}{t^{3/2}} = \frac{1}{\sqrt{2}}.\int t^{-3/2}\,dt = \frac{t^{-3/2+1}}{\sqrt{2}(-3/2+1)}.$$

$$\frac{t^{-1/2}}{\sqrt{2}(-1/2)} = \frac{t^{-1/2}}{-\sqrt{2}} = \frac{-2}{\sqrt{2}.t^{1/2}} = -\frac{2}{\sqrt{2}.\sqrt{t}} = -\frac{2}{\sqrt{2t}} + c$$

62. $\int \sqrt[3]{2-3x}.dx$

$$\int (2-3x)^{1/3}.\,dx\,.$$

$$\left\{\begin{matrix} v = (2-3x) \\ dv = -3\,dx \\ n = 1/3 \end{matrix}\right\}$$ El diferencial esta incompleto, falta (- 3). Se aplica: $\int v^n = \frac{v^{n+1}}{n+1} + c$.

$$\left(\frac{-1}{3}\right)\int (2-3x)^{1/3}(-3).\,dx = -\frac{(2-3x)^{1/3+1}}{3(1/3+1)} = -\frac{(2-3x)^{4/3}}{3(4/3)} =$$

$$\frac{-(2-3x)^{4/3}}{12/3} = -\cancel{3}\frac{(2-3x)^{4/3}}{\cancel{12}} = -\frac{(2-3x)^{4/3}}{4} + c\,.$$

63. $\int \frac{\text{sen } 2\theta \; d\theta}{\sqrt{\cos 2\theta}}$

$\int (\cos 2\theta)^{-1/2}.\text{sen } 2\theta \ d\theta$

$\left\{\begin{array}{l} v = (\cos 2\theta) \\ dv = -2 \text{ sen } 2\theta \ d\theta \\ n = -1/2 \end{array}\right\}$ Falta (-2) para completar el diferencial. Se aplica: $\int v^n = \frac{v^{n+1}}{n+1} + c$.

$\left(\frac{-1}{2}\right) \int (\cos 2\theta)^{-1/2}.(-2)\text{sen } 2\theta \ d\theta$

$\left(\frac{-1}{2}\right).\frac{(\cos 2\theta)^{-1/2+1}}{-1/2+1} = -\frac{(\cos 2\theta)^{1/2}}{2(1/2)} = -\frac{(\cos 2\theta)^{1/2}}{1} = -\sqrt{\cos 2\theta} + c.$

64. $\int \frac{e^x dx}{\sqrt{e^x - 5}}$.

$\int (e^x - 5)^{-1/2} . e^x dx$. $\left\{\begin{array}{l} v = (e^x - 5) \\ dv = e^x dx \\ n = -1/2 \end{array}\right\}$ El diferencial esta completo, se procede a integrar.

$\int (e^x - 5)^{-1/2}.e^x dx = \frac{(e^x - 5)^{-1/2+1}}{-1/2+1} = \frac{(e^x - 5)^{1/2}}{1/2} = 2(e^x - 5)^{1/2} + c$

65. $\int \frac{2 \ dx}{\sqrt{3 + 2x}}$.

$\int (3 + 2x)^{-1/2}. \ 2 \ dx$

$\left.\begin{array}{l} v = (3 + 2x) \\ dv = 2 \ dx \\ n = -1/2 \end{array}\right\}$ El diferencial esta completo , se procede a integrar.

$\int (3 + 2x)^{-1/2}. \ 2dx = \frac{(3 + 2x)^{-1/2+1}}{-1/2+1} = \frac{(3 + 2x)^{1/2}}{1/2} = 2(3 + 2x)^{1/2} =$

$2\sqrt{(3+2x)} + c$

66. $\int \frac{3\,dx}{2+3x} =$

$\left.\begin{array}{l} v = 2+3x \\ dv = 3\,dx \end{array}\right\}$ El diferencial esta completo, se usa la fórmula: $\int \frac{dv}{v} = \ln v + c$.

$\int \frac{3\,dx}{2+3x} = \ln(2+3x) + c$.

67. $\int \frac{x\,dx}{\sqrt{1-2x^2}}$.

$\int (1-2x^2)^{-1/2}.\ x\,dx$.

$\left.\begin{array}{l} v = (1-2x^2) \\ dv = -4x\,dx \\ n = -1/2 \end{array}\right\}$ El diferencial esta incompleto, falta (- 4) y se le opone (-1/4) .

$\left(\frac{-1}{4}\right)\int (1-2x^2)^{-1/2}.(-4)\,x\,dx = \frac{-1}{4}.\ \frac{(1-2x^2)^{-1/2+1}}{-1/2+1}$

$\frac{-(1-2x^2)^{1/2}}{4(1/2)} = -\frac{(1-2x^2)^{1/2}}{2} + c$.

68. $\int \frac{t\,dt}{3t^2+4}$.

$\left.\begin{array}{l} v = 3t^2+4 \\ dv = 6t\,dt \end{array}\right\}$ El diferencial esta incompleto, falta (6) y se le opone (1/6) .

$\left(\frac{1}{6}\right)\int \frac{(6)t\,dt}{3t^2+4} = \frac{1}{6}.\ln(3t^2+4) = \frac{\ln(3t^2+4)}{6} + c$.

69. $\int \left\{ \sqrt{x} - \dfrac{1}{\sqrt{x}} \right\}^2$

$$\int \left\{ \left(\sqrt{x}\right)^2 - 2\sqrt{x}.\dfrac{1}{\sqrt{x}} + \left\{ \dfrac{1}{\sqrt{x}} \right\}^2 \right\} dx = \int \left\{ x - 2 + \left\{ \dfrac{1^2}{\left(\sqrt{x}\right)^2} \right\} \right\} dx$$

70. $\int \left\{ y^2 - \dfrac{1}{y^2} \right\}^3 .dy$

$$\int \left\{ (y^2)^3 - 3\left((y^2)^2 . \dfrac{1}{y^2} \right) + 3\left((y^2) . \dfrac{1}{y^2} \right)^2 - \dfrac{1}{y^2} \right\}^3 . dy \quad .$$

$$\int \left\{ y^6 - \dfrac{3.\cancel{y^2}.y^2}{\cancel{y^2}} + \dfrac{3.\cancel{y^2}}{\cancel{y^2}.y^2} - \dfrac{1}{y^6} \right\} dy = \int \left\{ y^6 - 3y^2 + \dfrac{3}{y^2} - \dfrac{1}{y^6} \right\} dy.$$

$$\dfrac{y^{6+1}}{6+1} - 3.\dfrac{y^{2+1}}{2+1} + 3\int y^{-2}\,dy - \int y^{-6}\,dy =$$

$$\dfrac{y^7}{7} - \dfrac{\cancel{3}y^3}{\cancel{3}} + \dfrac{3.y^{-2+1}}{-1} - \dfrac{y^{-6+1}}{-5} =$$

$$\dfrac{y^7}{7} - y^3 - 3.y^{-1} + \dfrac{y^{-5}}{5} = \dfrac{y^7}{7} - y^3 - \dfrac{3}{y} + \dfrac{1}{5y^5} + c\,.$$

71. $\int \dfrac{\textbf{sen } a\theta \ d\theta}{\textbf{cos } a\theta}$

Según Trigonometría: $\dfrac{\text{sen } a\theta}{\cos a\theta} = \text{tg } a\theta$. □ $\int \text{tg } a\theta.\ d\theta$.

$$\left.\begin{array}{l} v = a\theta \\ dv = a\,d\theta \end{array}\right\} \begin{array}{l} \text{Utilizamos la integral:} \\ \int \text{tg } v\ dv = -\ln \cos v = \ln \sec v + c\,. \end{array}$$

$$\left(\dfrac{1}{a}\right)\int \text{tg } a\theta.\ (a)d\theta = -\left\{\dfrac{\ln \cos (a\theta)}{a}\right\} = \dfrac{\ln \sec (a\theta)}{a} + c\,.$$

72. $$\int \frac{\csc^2\varphi \; d\varphi}{\sqrt{(2\cot\varphi + 3)}} .$$

$$\int (2\cot\varphi + 3)^{-1/2} . \csc^2\varphi \; d\varphi .$$

$v = (2\cot\varphi + 3)$
$dv = -2\csc^2\varphi \; d\varphi$

Falta (-2) para completar el diferencial.
Se aplica: $\int \mathbf{v^n\, dv} = \frac{\mathbf{v^{n+1}}}{n+1} + \mathbf{c}$

$$\frac{-1}{2}\int (2\cot\varphi + 3)^{-1/2}.(-2)\csc^2\varphi \; .d\varphi = -\frac{1}{2}.\frac{(2\cot\varphi + 3)^{-1/2+1}}{-1/2+1} =$$

$$-\frac{1}{2}.\frac{(2\cot\varphi + 3)^{1/2}}{1/2} = -\frac{(2\cot\varphi + 3)^{1/2}}{2(1/2)} = -\frac{(2\cot\varphi + 3)^{1/2}}{1} =$$

$$-(2\cot\varphi + 3)^{1/2} = -\sqrt{(2\cot\varphi + 3)} + c .$$

73. $$\int \frac{(2x+5)\,dx}{x^2+5x+6}$$

$v = x^2 + 5x + 6$
$dv = (2x+5) . dx$

El diferencial esta completo,
aplicamos la fórmula: $\int dv/v = \ln v + c$.

$$\int \frac{(2x+5)\,dx}{x^2+5x+6} = \ln\,(2x+5) + c .$$

74. $$\int \frac{(2x+7)\,dx}{x+3}$$

Dividimos:

$$\begin{array}{r|l} 2x+7 & x+3 \\ \hline -2x-6 & 2 \\ \hline +1 & \end{array}$$

El resultado es: $2 + \frac{1}{x+3}$.

$$\int \left(2 + \frac{1}{x+3} \right) dx \ .$$

$$2 \int dx + \int \frac{dx}{x+3} = 2x + \ln(x+3) + c \ .$$

75. $\displaystyle \int \frac{(x^2+2)\,dx}{x+2}$

Dividimos:

$$\begin{array}{rrr|l} x^2 & & +2 & x+2 \\ \hline -x^2 & -2x & & x-2 \\ & -2x & +2 & \\ & +2x & +4 & \\ \hline & & +6 & \end{array}$$

El resultado es:

$$x - 2 + \left(\frac{6}{x+2} \right)$$

$$\int \left[x - 2 + \frac{6}{x+2} \right] dx = \int x\,dx - 2 \int dx + 6 \int \frac{dx}{x+2} =$$

$$\frac{x^2}{2} - 2x + 6 \ln(x+2) + c.$$

76. $\displaystyle \int \frac{(x^3+3x)\,dx}{x^2+1}$

Dividimos:

$$\left\{ \begin{array}{rr|l} x^3 & +3x & x^2+1 \\ \hline -x^3 & -x & x \\ \hline & +2x & \end{array} \right\}$$

El resultado de la división es :

$$\left(x + \frac{2x}{x^2+1} \right)$$

$$\left. \begin{array}{l} v = x^2+1 \\ dv = 2x\,dx \end{array} \right\} \begin{array}{l} \text{El diferencial esta completo} \\ \text{se procede a integrar.} \end{array}$$

$$\int x\,dx + \int \frac{2x\,dx}{x^2+1} = \frac{x^{1+1}}{1+1} + \ln(x^2+1) = \frac{x^2}{2} + \ln(x^2+1) + c \ .$$

77. $\int \dfrac{(4x + 3)\,dx}{\sqrt[3]{1 + 3x + 2x^2}}$.

$\int (1 + 3x + 2x^2)^{-1/3}.(4x + 3)\,dx$.

$\left\{\begin{array}{l} v = (1 + 3x + 2x^2) \\ dv = 3 + 4x\,dx = 4x + 3\,dx \\ n = -1/3 \end{array}\right\}$ El diferencial esta completo, se procede a integrar.

$\int (1 + 3x + 2x^2)^{-1/3} . (4x + 3)\,dx = \dfrac{(1 + 3x + 2x^2)^{-1/3+1}}{-1/3 + 1}$.

$\dfrac{(1 + 3x + 2x^2)^{2/3}}{2/3} = \dfrac{3\,(1 + 3x + 2x^2)^{2/3}}{2} + c$.

78. $\int \dfrac{(e^t + 2)\,dt}{e^t + 2t}$

$\left.\begin{array}{l} v = e^t + 2t \\ dv = (e^t + 2)\,dt \end{array}\right\}$ El diferencial esta completo. Se aplica: $\int dv/v = \ln v + c$.

$\int \dfrac{(e^t + 2)\,dt}{e^t + 2t} = \ln(e^t + 2t) + c$.

79. $\int \dfrac{(e^x + \operatorname{sen} x)\,dx}{\sqrt{e^x - \cos x}}$

$\int (e^x - \cos x)^{-1/2}.(e^x + \operatorname{sen} x)\,dx$

$\left.\begin{array}{l} v = (e^x - \cos x) \\ dv = (e^x - (-\operatorname{sen} x)\,dx = (e^x + \operatorname{sen} x)\,dx \\ n = -1/2 \end{array}\right\}$ El diferencial esta completo,se procede a integrar.

$$\frac{(e^x - \cos x)^{-1/2+1}}{-1/2+1} = \frac{(e^x - \cos x)^{1/2}}{1/2} = 2(e^x - \cos x)^{1/2} + c\ .$$

80. $\displaystyle\int \frac{\sec 2\theta \ \ \operatorname{tg} 2\theta \ \ d\theta}{3 \sec 2\theta \ - 2}$

$$\left.\begin{aligned} v &= 3 \sec 2\theta - 2 \\ dv &= 3\{\sec 2\theta \ .\ \operatorname{tg} 2\theta\}.2\ d\theta = \\ dv &= \{6 \sec 2\theta \ .\ \operatorname{tg} 2\theta\}\ d\theta \end{aligned}\right\}$$

Falta (6) para completar el diferencial y se le opone (1/6). Se aplica: $\int dv/v = \ln v + c$.

$$\left(\frac{1}{6}\right)\int \frac{(6)\sec 2\theta \ \ \operatorname{tg} 2\theta \ \ d\theta}{3 \sec 2\theta - 2} = \frac{1}{6}.\ \ln (3 \sec 2\theta - 2) =$$

$$\frac{\ln (3 \sec 2\theta - 2)}{6} + c\ .$$

81. $\displaystyle\int \frac{\sec^2 2t\ dt}{\sqrt{5 + 3\operatorname{tg} 2t}}\ .$

$\int (5 + 3\operatorname{tg} 2t)^{-1/2}.\sec^2 2t\ dt$.

$$\left.\begin{aligned} v &= (5 + 3\operatorname{tg} 2t) \\ dv &= 3(\sec^2 2t)(2)\ dt \\ dv &= 6 \sec^2 2t\ dt \\ n &= -1/2 \end{aligned}\right\}$$

Falta (6)para completar el diferencial . Se aplica: $\int \mathbf{v^n\ dv} = \dfrac{\mathbf{v^{n+1}}}{\mathbf{n+1}} + \mathbf{c}$

$$\left(\frac{1}{6}\right)\int (5 + 3\operatorname{tg} 2t)^{-1/2}.(6)\sec^2 2t\ dt$$

$$\left(\frac{1}{6}\right).\ \frac{(5 + 3\operatorname{tg} 2t)^{-1/2+1}}{-1/2+1} = \frac{(5 + 3\operatorname{tg} 2t)^{1/2}}{6(1/2)} = \frac{(5 + 3\operatorname{tg} 2t)^{1/2}}{3} + c\ .$$

Problemas. Pagina 241

Verificar las Siguientes Integraciones:

1. $\int 6\, e^{3x}\, dx = 2\, e^{3x} + c$.

$6 \int e^{3x}\, dx$

$v = 3x$
$dv = 3\, dx$

Falta el (3) para completar el diferencial,
luego se procede a integrar.
Se aplica: $\int e^{v}\, dv = e^{v} + c$.

$\cancel{6}\left(\frac{1}{\cancel{3}}\right) \int e^{3x}.(3)\, dx = 2\, e^{3x} + c$.

2. $\int e^{x/n}\, dx = n e^{x/n} + c$.

$v = x/n$

Falta 1/n completar en el diferencial,

$dv = 1/n$ luego se procede a integrar. Se aplica:
$\int e^v \, dv = e^v + c$.

$(n) \int e^{x/n} .(1/n) \, dx = n.e^{x/n} + c$.

3. $\int \frac{dx}{e^x} = -\frac{1}{e^x} + c$.

$\int e^{-x}. \, dx$; $\{ v = -x \; ; \; dv = -dx \}$

Para completar el diferencial, le falta el signo (-).

$(-) \int e^{-x}.(-) \, dx = - e^{-x} = - \frac{1}{e^x} + c$.

4. $\int 10^x \, dx = \frac{10^x}{\ln 10} + c$.

$\left.\begin{array}{l} v = x \\ dv = dx \end{array}\right\}$ El diferencial esta completo, se usa la fórmula:
$\int a^v \, dv = \frac{a^v}{\ln a} + c$.

$\int 10^x \, dx = \frac{10^x}{\ln 10} + c$.

5. $\int a^{ny} \, dy = \frac{a^{ny}}{n \ln a} + c$.

$\left.\begin{array}{l} v = ny \\ dv = n.dy \end{array}\right\}$ Falta (n) para completar el diferencial.
Se aplica: $\int a^v \, dv = \frac{a^v}{\ln a} + c$.

$(1/n) \int a^{ny}.(n) \, dy = \frac{1}{n} . \frac{a^{ny}}{\ln a} = \frac{a^{ny}}{n \ln a} + c$.

6. $\displaystyle\int \frac{e^{\sqrt{x}}\,dx}{\sqrt{x}} = 2e^{\sqrt{x}} + c.$

$$\int e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} \left\{\frac{1}{2}\right\} \cdot dx =$$

$$\left.\begin{array}{l} v = \sqrt{x} \\ dv = \dfrac{1}{2\sqrt{x}} \cdot dx \end{array}\right\}$$

Falta (1/2) para completar el diferencial, luego se procede a integrar. Se aplica: $\int e^v\,dv = e^v + c$.

$$\int e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} \left\{\frac{1}{2}\right\} \cdot dx = (2)\int e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \cdot dx = 2e^{\sqrt{x}} + c.$$

7. $\int (e^{x/a} + e^{-x/a})\,dx = a\,(e^{x/a} - e^{-x/a}) + c.$

$$\int e^{x/a}\,dx + \int e^{-x/a}\,dx. \qquad \left\{\begin{array}{l} v = x/a \\ dv = 1/a\,dx \end{array}\right\} \left\{\begin{array}{l} v = -x/a \\ dv = -1/a\,dx \end{array}\right\}$$

Una vez completado los diferenciales, se integra.

$$(a)\int e^{x/a}.(1/a)\,dx + (-a)\int e^{-x/a}.(-1/a)\,dx$$

$$a.e^{x/a} - a.e^{-x/a} = a\,(e^{x/a} - e^{-x/a}) + c.$$

8. $\int (e^{x/a} - e^{-x/a})^2\,dx$

Desarrollando el producto notable: $(e^{x/a} - e^{-x/a})^2$:

$$(e^{x/a} - e^{-x/a})^2 = \{(e^{x/a})^2 - 2(e^{x/a})(e^{-x/a}) + (e^{-x/a})^2\}.$$

$$e^{2x/a} - 2e^{+x/a-x/a} + e^{-2x/a} = e^{2x/a} - 2e^{0} + e^{-2x/a}.$$

$$e^{2x/a} - 2(1) + e^{-2x/a} = e^{2x/a} - 2 + e^{-2x/a}.$$

Sustituyendo : $\{e^{2x/a} - 2 + e^{-2x/a}\}$ en la integral .

$\int \{e^{2x/a} - 2 + e^{-2x/a}\}\, dx = \int e^{2x/a}\, dx - 2\int dx + \int e^{-2x/a}\, dx$.

Completando el diferencial, antes de integrar :

$\left\{\begin{array}{l} v = 2x/a \\ dv = 2/a\, dx \end{array}\right\} \quad \left\{\begin{array}{l} v = -2x/a \\ dv = -2/a\, dx \end{array}\right\}$

Se aplica en ambas integrales: $\int e^v\, dv = e^v + c$.

$(a/2)\int e^{2x/a}.(2/a)\, dx - 2\int dx + (-a/2)\int e^{-2x/a}.(-2/a)\, dx$.

$\frac{a}{2}.e^{2x/a} - 2x - \frac{a}{2}.e^{-2x/a} = \frac{a}{2}.\{e^{2x/a} - e^{-2x/a}\} - 2x + c$.

9. $\int x\, e^{x^2}\, dx = \frac{1}{2}.e^{x^2} + c$.

$\left.\begin{array}{l} v = x^2 \\ dv = 2x\, dx \end{array}\right\}$ Como el diferencial esta completo, se procede a integrar.

$\int x\, e^{x^2}\, dx = \frac{1}{2}.e^{x^2} + c$.

10. $\int e^{\operatorname{sen} x} \cos x\, dx = e^{\operatorname{sen} x} + c$.

$\left.\begin{array}{l} v = \operatorname{sen} x \\ dv = \cos x\, dx \end{array}\right\}$ El diferencial esta completo, se procede a integrar.

$\int e^{\operatorname{sen} x}. \cos x\, dx = e^{\operatorname{sen} x} + c$.

11. $\int e^{\operatorname{tg}\theta} \sec^2\theta\, d\theta$.

$v = tg\,\theta$; $dv = sec^2\theta\ d\theta$ } El diferencial esta completo, se procede a integrar.

$\int e^{tg\,\theta}.\ sec^2\theta\ d\theta = e^{tg\,\theta} + c$.

12. $\int \sqrt{e^t}\ dt = 2\sqrt{e^t} + c.$

$\int (e^t)^{1/2}\,dt = \int e^{t/2}.\,dt$ { $v = t/2$; $dv = 1/2$ } Falta (1/2) en el diferencial, luego se procede a integrar.

Se aplica: $\int e^v\,dv = e^v + c$.

$(2)\int e^{t/2}.(1/2)\,dt = 2e^{t/2} + c$.

13. $\int a^x e^x\,dx$

´-0

$$\left.\begin{array}{l} v = a^x\, e^x \\ dv = \{a^x.e^x + e^x.\ a^x.\ln a\}\ dx \\ dv = a^x.e^x\{1 + \ln a\}\ dx \end{array}\right\}$$ Falta (1 + ln a) para completar el diferencial, luego se procede a integrar.

$$\left(\frac{1}{1+\ln a}\right).\int a^x\, e^x.(\,1 + \ln a)\ dx = \frac{a^x e^x}{1 + \ln a} + c\ .$$

14. $\int a^{2x}\, dx = \dfrac{a^{2x}}{2 \ln a} + c\ .$

$$\left.\begin{array}{l} v = 2x \\ dv = 2\ dx \end{array}\right\}$$ Falta (2) para completar el diferencial. Se aplica: $\int a^v\, dv = \dfrac{a^v}{\ln a} + c\ .$

$$\left(\frac{1}{2}\right)\int a^{2x}.(2)\ dx = \frac{1}{2}.\ \frac{a^{2x}}{\ln a} = \frac{a^{2x}}{2 \ln a} + c\ .$$

15. $\int (e^{5x} + a^{5x})\ dx = \dfrac{1}{5}\left\{e^{5x} + \dfrac{a^{5x}}{\ln a}\right\} + c\ .$

$$\int e^{5x}.\ dx + \int a^{5x}.\ dx$$

Completando los diferenciales de ambas integrales.

$$\begin{Bmatrix} v = 5x \\ dv = 5\ dx \end{Bmatrix} \quad \begin{Bmatrix} v = 5x \\ dv = 5\ dx \end{Bmatrix}$$

Se aplica: $\int e^v\, dv = e^v + c$.

$(1/5)\int e^{5x}.(5)\, dx + (1/5)\int a^{5x}.(5)\, dx$

$$\frac{1}{5}.e^{5x} + \frac{1}{5}.\frac{a^{5x}}{\ln a} = \frac{1}{5}\left\{e^{5x} + \frac{a^{5x}}{\ln a}\right\} + c\,.$$

16. $\int 5e^{ax}\, dx$

$\left.\begin{matrix} v = ax \\ dv = a\ dx \end{matrix}\right\}$ Falta (a) para completar el diferencial, luego se procede a integrar.

Se aplica: $\int e^v\, dv = e^v + c$.

$$5\left(\frac{1}{a}\right)\int e^{ax}.(a)\, dx = \frac{5e^{ax}}{a} + c\,.$$

17. $\int \frac{3\,dx}{e^x}$

$3 \int e^{-x}.\,dx$

$v = -x$
$dv = -dx$ } Falta el signo (-) , para completar el diferencial, luego se procede a integrar.

Se aplica: $\int e^v\,dv = e^v + c$.

$3(-)\int e^{-x}.(-)\,dx = -3.e^{-x} = -\frac{3}{e^x} + c$.

18. $\int \frac{4\,dt}{\sqrt{e^t}} =$

$\int (e^t)^{-1/2}\,dt = 4(-2)\int e^{-t/2}.(-1/2)\,dt =$

$-8\,e^{-t/2} = -\frac{8}{e^{t/2}} + c$.

19. $\int c^{ax}\,dx$

Suponemos que : "c" de la integral dada es la constante "a" de

la formula.

$v = ax$
$dv = a\,dx$ } Falta (a) para completar el diferencial, luego se procede a integrar.

Empleando la fórmula: $\int a^v.\,dv = \frac{a^v}{\ln a} + c$

$(1/a)\int c^{ax}.(a)\,dx = .\frac{1}{a}.\frac{c^{ax}}{\ln c} + c$.

20. $\int \frac{dx}{4^{2x}}$.

$\int 4^{-2x}.\, dx$

$\left.\begin{array}{l} v = -2x \\ dv = -2\, dx \end{array}\right\}$ Falta (- 2) , para completar el diferencial, luego se procede a integrar.

Utilizamos la fórmula: $\int a^v.\, dv = \dfrac{a^v}{\ln a} + c$

$$(-1/2)\int 4^{-2x}.(-2)\, dx = .\frac{-1}{2}.\frac{4^{-2x}}{\ln 4} = \frac{-1}{2 . \ln 4 . 4^{2x}} + c .$$

21. $\int x^2 e^{x^3}\ dx$

Ordenando: $\int e^{x^3}.\ x^2\ dx$

$\left.\begin{array}{l} v = x^3 \\ dv = 3x^2\, dx \end{array}\right\}$ Falta (3) para completar el diferencial, luego se procede a integrar.

Se aplica: $\int e^v.\, dv = e^v + c$.

$$(1/3)\int e^{x^3}\ .(3)\, x^2\, dx = .\frac{1}{3}.e^{x^3} = \frac{e^{x^3}}{3} + c$$

22. $\int \dfrac{(e^x + 4)\, dx}{e^x}$

$$\int \frac{e^x}{e^x} dx + 4\int \frac{dx}{e^x} = \int dx + 4(-)\int e^{-x}.(-)\, dx = x - 4e^{-x} = x - \frac{4}{e^x} + c .$$

23. $\int \dfrac{e^x\, dx}{e^x - 2}$

$$\left.\begin{array}{l} v = e^x - 2 \\ dv = e^x \, dx \end{array}\right\} \text{ El diferencial esta completo, aplicamos : } \int \frac{dv}{v} = \ln v + c .$$

□ $\ln(e^x - 2) + c$.

24. $\int x\,(e^{x^2} + 2)\,dx$

$$\int \{(e^{x^2} + 2) \,.\, x\}\, dx$$

$$\int e^{x^2} . \, x\, dx + 2\int x\, dx$$

$$\left.\begin{array}{l} v = x^2 \\ dv = 2x\, dx \end{array}\right\}$$ Falta (2) en la 1^{ra} integral, para completar el diferencial , el 2^{do} integral esta completo.

Se aplica: $\int e^v\, dv = e^v + c$, en la 1^{ra} integral .

$$(1/2)\int e^{x^2} .(2)\, x\, dx + 2\int x\, dx = \frac{1}{2} . e^{x^2} + 2 . \frac{x^{1+1}}{1+1} =$$

$$\frac{e^{x^2}}{2} + \frac{\cancel{2} . x^2}{\cancel{2}} = \frac{e^{x^2}}{2} + x^2 + c.$$

25. $\int \frac{(e^{\sqrt{x}} - 3)\, dx}{\sqrt{x}}$

$$\int e^{\sqrt{x}} . \frac{1}{\sqrt{x}} . \, dx - 3\int \frac{dx}{\sqrt{x}} .$$

$$\left.\begin{array}{l} v = \sqrt{x} \\ dv = \frac{1}{2} . \frac{1}{\sqrt{x}} . \, dx \end{array}\right\}$$ Falta (1/2) para completar el diferencial, de la 1^{ra} integral. Se aplica: $\int e^v\, dv = e^v + c$.

$(2)\int e^{\sqrt{x}} . \frac{1}{2} . \frac{1}{\sqrt{x}} . dx \; - 3\int x^{-1/2} dx = 2e^{\sqrt{x}} - \frac{3.x^{-1/2+1}}{-1/2+1} =$

$2e^{\sqrt{x}} - \frac{3.x^{1/2}}{1/2} = 2e^{\sqrt{x}} - 6x^{1/2} = 2e^{\sqrt{x}} - 6\sqrt{x} + c$.

26. $\int t\, 2^{t2}\, dt$

$\int 2^{t2} . t\, dt$

$v = t^2$
$dv = 2t\, dt$

Falta (2) para completar el diferencial, luego se procede a integrar.
Se aplica: $\int a^v . dv = \frac{a^v}{\ln a} + c$

$(1/2)\int 2^{t2} .(2)\, t\, dt = \frac{1}{2} . \frac{2^{t2}}{\ln 2} = \frac{2^{t2}}{2 \ln 2} + c$.

27. $\int \frac{a\, d\theta}{b^{3\theta}}$

$a\int b^{-3\theta} . d\theta$

$v = -3\theta$
$dv = -3d\theta$

Falta (- 3) para completar el diferencial.
Se aplica: $\int a^v dv = a^v/\ln a + c$.

$a(-1/3)\int b^{-3\theta}.(-3)\, d\theta = \frac{-a}{3} . \frac{b^{-3\theta}}{\ln b} = \frac{-a}{(3 \ln b)\, b^{3\theta}} + c.$

28. $\int 6\, x\, e^{-x2}\, dx$

Descomponiendo el # 6 en 2 factores y ordenando:

$3\int e^{-x^2}.2x \; dx$

$\left.\begin{array}{l} v = -x^2 \\ dv = -2x\,dx \end{array}\right\}$ Falta el signo (-) para completar el diferencial. Se aplica: $\int e^v\,dv = e^v + c$.

$3(-)\int e^{-x^2}.(-)2x\,dx = -3e^{-x^2} = \dfrac{-3}{e^{x^2}} + c$.

29. $\int (e^{2x})^2\,dx$

$\int e^{4x}\,dx$

$\left.\begin{array}{l} v = 4x \\ dv = 4\,dx\,. \end{array}\right\}$ Falta el # 4 para completar el diferencial. Se aplica: $\int e^v\,dv = e^v + c$.

$(1/4)\int e^{4x}.(4)\,dx = .\dfrac{1}{4}.e^{4x} = \dfrac{e^{4x}}{4} + c$.

30. $\int \dfrac{x^2\,dx}{e^{x^3}}$

$\int e^{-x^3}.\,x^2\,dx$

$\left.\begin{array}{l} v == -x^3 \\ dv = -3x^2\,dx \end{array}\right\}$ Falta (- 3) para completar el diferencial. Se aplica: $\int e^v\,dv = e^v + c$.

$\dfrac{-1}{3}\int e^{-x^3}.(-3)\,x^2\,dx = \dfrac{-1}{3}.\,e^{-x^3} = \dfrac{-1}{3\,e^{x^3}} + c$.

Problemas. Paginas 244 y 245

Verificar las siguientes Integraciones:

1. $\int \cos mx\, dx = \frac{1}{m} \operatorname{sen} mx + c$.

$$\left.\begin{array}{l} v = mx \\ dv = m\, dx \end{array}\right\} \begin{array}{l} \text{Falta (m) para completar el diferencial.} \\ \text{Se aplica: } \int \cos v\, dv = \operatorname{sen} v + c \text{ .} \end{array}$$

$\left(\frac{1}{m}\right) \int \cos mx\, .(m)\, dx = \frac{1}{m} \operatorname{sen} mx + c$.

2. $\int \operatorname{tg} bx\, dx = \frac{1}{b} \ln \sec bx + c$.

$$\left.\begin{array}{l} v = bx \\ dv = b\, dx \\ \end{array}\right\} \begin{array}{l} \text{Falta (b) para completar el diferencial.} \\ \text{Se aplica:} \\ \int \operatorname{tg} x\, dx = -\ln \{\cos (v)\} + c = \ln \{\sec (v)\} + c \text{ .} \end{array}$$

$\left(\frac{1}{b}\right) \int \operatorname{tg} bx\, .(b)\, dx = \frac{1}{b} \ln \sec bx + c$.

3. $\int \sec ax\, dx = \frac{1}{a} \ln (\sec ax + \operatorname{tg} ax) + c$.

$\left.\begin{array}{l} v = ax \\ dv = a\,dx \end{array}\right\}$ Falta (a) para completar el diferencial. Usamos la fórmula: $\int \sec v\,dv = \ln(\sec v + \operatorname{tg} v) + c.$

$\left(\frac{1}{a}\right)\int \sec ax\,.(a)\,dx = \frac{1}{a}\ln(\sec ax + \operatorname{tg} ax) + c\,.$

4. $\int \csc v\,dv = \ln \operatorname{tg} \frac{1}{2} v + c\,.$

$\ln(\csc v - \cot v) = \ln\left(\frac{1}{\operatorname{sen} v} - \frac{\cos v}{\operatorname{sen} v}\right) = \ln\left(\frac{1 - \cos v}{\operatorname{sen} v}\right) =$

$\ln \operatorname{tg} \frac{1}{2} v + c\,.$

Por trigonometría :

$\csc v = \frac{1}{\operatorname{sen} v}$; $\cot v = \frac{\cos v}{\operatorname{sen} v}$; $\operatorname{tg} \frac{v}{2} = \frac{1 - \cos v}{\operatorname{sen} v}.$

 Esta demostrado : $\int \csc v\,dv = \ln \operatorname{tg} \frac{1}{2} v + c\,.$

5. $\int \sec 3t \operatorname{tg} 3t\,dt = \frac{1}{3}\sec 3t + c\,.$

$\left.\begin{array}{l} v = 3t \\ dv = 3\,dt \end{array}\right\}$ Falta (3) para completar el diferencial. Se aplica: $\int \sec v \operatorname{tg} v\,dv = \sec v + c\,.$

$(1/3)\int \sec 3t\,.\operatorname{tg} 3t\,(3)\,dt = \frac{1}{3}\sec 3t + c\,.$

$.\frac{1}{3}.\{\sec 3t\} + c\,.$

6. $\int \csc ay \cot ay\, dy = \frac{-1}{a} \csc ay + c$

$$\left.\begin{array}{l} v = ay \\ dv = a\,dy \end{array}\right\} \begin{array}{l} \text{Falta (a) para completar el diferencial. Se aplica:} \\ \int \csc v \cot v\, dv = -\csc v + c \end{array}$$

$(1/a) \int \csc ay \,.\, \cot ay.\ (a)\, dy\ .$

$.\frac{1}{a}.\{ -\csc ay \} = \frac{-1}{a} \csc ay + c\ .$

7. $\int \csc^2 3x\, dx = \frac{-1}{3} \cot 3x + c\ .$

$$\left.\begin{array}{l} v = 3x \\ dv = 3\,dx \end{array}\right\} \begin{array}{l} \text{Completando el diferencial con (3) .} \\ \text{Se aplica: } \int \csc^2 v\, dv = -\cot v + c\ . \end{array}$$

$(1/3) \int \csc^2 3x\ .\ (3)\, dx = \frac{1}{3} \{-\cot 3x\} = \frac{-1}{3} \cot 3x + c\ .+ c\ .$

8. $\int \cot \frac{x}{2}\, dx$

$$\left.\begin{array}{l} v = \frac{1}{2}x \\ \\ dv = \frac{1}{2}\,dx \end{array}\right\} \begin{array}{l} \text{Falta (1/2) para completar el diferencial.} \\ \text{Se aplica:} \\ \int \cot v\, dv = \ln \{\text{sen}\,(v)\} + c\ . \end{array}$$

$(2) \int \cot \frac{x}{2} \left(\frac{1}{2}\right) dx = 2 \ln \left(\text{sen} \frac{x}{2}\right) + c\ .$

9. $\int x \sec^2 x^3 = \frac{1}{3}.\ \text{tg}\, x^3 + c\ .$

Ordenando: $\int (\sec x^3)^2 \,.\, x\, dx = \int \sec^2 x^3 \,.\, x\, dx$

$\left.\begin{array}{l} v = x^3 \\ dv = 3x^2\, dx \end{array}\right\}$ Falta (3) para completar el diferencial. Se aplica: $\int \sec^2 v \,.\, dv = \operatorname{tg} v + c$.

$\frac{1}{3} . \int (\sec x^3)^2 .(3)\, x\, dx =$

$\frac{1}{3} . \operatorname{tg} x^3 + c$.

10. $\int \frac{dx}{\operatorname{sen}^2 x}$.

Por Trigonometría: $\frac{1}{\operatorname{sen}^2 x} = \csc^2 x$

$\int \csc^2 x \;\; dx = -\cot^2 x + c$.

11. $\int \frac{ds}{\cos^2 s} = \operatorname{tg} s + c$.

Por Trigonometría: $\frac{1}{\cos^2 s} = \sec^2 s$

$\int \sec^2 s\, ds = \operatorname{tg} s + c$.

12. $\int (\operatorname{tg}\theta + \cot\theta)^2\, d\theta = \operatorname{tg}\theta - \cot\theta + c$.

$\int (\operatorname{tg}^2\theta + 2\operatorname{tg}\theta\cot\theta + \cot^2\theta)\, d\theta =$

$\left\{\begin{array}{l} \text{Por Trigonometría:} \\ \operatorname{tg}\theta \,.\, \cot\theta = 1 \;\; ; \operatorname{tg}^2\theta + 1 = \sec^2\theta \;\; ; \;\; \cot^2\theta + 1 = \csc^2\theta. \\ \text{Utilizando un artificio matemático : } 2 = 1 + 1 \text{ .} \end{array}\right\}$

Reemplazando y utilizando el artificio, obtenemos:

$\int (\operatorname{tg}^2 \theta + 2(1) + \cot^2 \theta)\, d\theta = \int (\operatorname{tg}^2 \theta + 2 + \cot^2 \theta)\, d\theta$

$\int (\operatorname{tg}^2 \theta + 1 + 1 + \cot^2 \theta)\, d\theta = \int (\operatorname{tg}^2 \theta + 1 + \cot^2 \theta + 1)\, d\theta$

Pero: $\operatorname{tg}^2 \theta + 1 = \sec^2 \theta$; $\cot^2 \theta + 1 = \csc^2 \theta$.

□ $\int \sec^2 \theta \, d\theta + \int \csc^2 \theta \, d\theta = \operatorname{tg} \theta - \cot \theta + c$.

13. $\int (\sec \varphi - \operatorname{tg} \varphi)^2 d\varphi = 2 (\sec \varphi - \operatorname{tg} \varphi) - \varphi + c$ **.**

$\int (\sec^2 \varphi - 2 \sec \varphi \operatorname{tg} \varphi + \operatorname{tg}^2 \varphi)\, d\varphi =$

Pero: $\operatorname{tg}^2 \varphi = \sec^2 \varphi - 1$, sustituyendo en la integral.
$\int (\sec^2 \varphi - 2 \sec \varphi \operatorname{tg} \varphi + \sec^2 \varphi - 1)\, d\varphi =$

$\int (2\sec^2 \varphi - 2 \sec \varphi \operatorname{tg} \varphi - 1)\, d\varphi =$

$\int 2\sec^2 \varphi \, d\varphi - 2 \int \sec \varphi \operatorname{tg} \varphi \, d\varphi - \int d\varphi =$

$2 \int \sec^2 \varphi \, d\varphi - 2 \int \sec \varphi \operatorname{tg} \varphi \, d\varphi - \int d\varphi =$

En la 1ª integral aplicamos: $\int \sec^2 v \, dv = \operatorname{tg} v + c$.

En la 2da integral aplicamos: $\int \sec v \operatorname{tg} v \, dv = \sec v + c$.

$2 \operatorname{tg} \varphi - 2\sec \varphi - \varphi = 2(\operatorname{tg} \varphi - \sec \varphi) - \varphi + c$.

14. $\int \frac{dx}{1 + \cos x} = - \cot x + \csc x + c$ **.**

Racionalizando: $\frac{1}{1 + \cos x}$.

$$\frac{1}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}=\frac{1-\cos x}{1-\cos^2 x}.$$

Pero: $1-\cos^2 x = \operatorname{sen}^2 x$.

$\int \frac{1-\cos x}{\operatorname{sen}^2 x}.\, dx$. Aplicando artificios aritméticos, Ejm:

Aplicando artificios aritméticos, Ejm:

$$\left\{\frac{8-6}{2}=\frac{8}{2}-\frac{6}{2}\right\} \square \quad \left\{\frac{1-\cos x}{\operatorname{sen}^2 x}=\frac{1}{\operatorname{sen}^2 x}-\frac{\cos x}{\operatorname{sen}^2 x}.\right\}$$

$$\int\left(\frac{1}{\operatorname{sen}^2 x}-\frac{\cos x}{\operatorname{sen}^2 x}\right)dx=\int\frac{dx}{\operatorname{sen}^2 x}-\int\frac{\cos x\, dx}{\operatorname{sen}^2 x}=$$

$$\int \csc^2 x\, dx-\int(\operatorname{sen} x)^{-2}.\cos x\, dx=$$

$$\left.\begin{array}{l} v=\operatorname{sen} x \\ dv=\cos x\, dx \end{array}\right\} \begin{array}{l} \text{En la } 1^{\text{ra}} \text{ aplicamos: } \int \csc^2 v\, dv=-\cot v+c\,. \\ \text{El diferencial de la } 2^{\text{da}} \text{ integral, esta completo.} \end{array}$$

$$\int \csc^2 x\, dx-\int(\operatorname{sen} x)^{-2}.\cos x\, dx=-\cot x-\frac{(\operatorname{sen} x)^{-2+1}}{-2+1}=$$

Por Trigonometría : $\frac{1}{\operatorname{sen} x}=\csc x$.

$$=-\cot x-\frac{(\operatorname{sen} x)^{-1}}{-1}=-\cot x+\frac{1}{\operatorname{sen} x}=-\cot x+\csc x+c\,.$$

15. $\displaystyle\int\frac{dx}{1+\operatorname{sen} x}=\operatorname{tg} x-\sec x+c\,.$

Racionalizando y efectuando artificios aritméticos :

$$\frac{1}{1+\operatorname{sen} x}\cdot\frac{1-\operatorname{sen} x}{1-\operatorname{sen} x}=\frac{1-\operatorname{sen} x}{1-\operatorname{sen}^2 x}=\frac{1-\operatorname{sen} x}{\cos^2 x}.$$

$$\frac{1 - \text{sen } x}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\text{sen } x}{\cos^2 x} = \sec^2 x - \frac{\text{senx}}{\cos^2 x} =$$

$$\int \sec^2 x \, dx - \int \frac{\text{sen } x \, dx}{\cos^2 x} = \int \sec^2 x \, dx - \int (\cos x)^{-2}.\ \text{sen } x \, dx$$

$v = \cos x$ | En la 1ra integral aplicamos: $\int \sec^2 v \, dv = \text{tg } v + c$

$dv = -\text{ sen } x \, dx$ | En la 2da integral aplicamos: $\int v^n \, dv = \frac{v^{n+1}}{n+1} + c$

$$\int \sec^2 x \, dx - (-) \int (\cos x)^{-2}.(-) \text{ sen } x \, dx =$$

$$\text{tg } x + \frac{(\cos x)^{-2+1}}{-2+1} = \text{tg } x + \frac{(\cos x)^{-1}}{-1} = \text{tg } x - \frac{1}{\cos x} =$$

$\text{tg } x - \sec x + c$.

16. $\displaystyle\int \frac{\text{sen } s \, ds}{1 + \cos s} = -\ln (1 + \cos s) + c$ **.**

$v = 1 + \cos s$ | Falta el signo (-) , para completar el diferencial

$dv = -\text{ sen } s \, ds$ | Aplicamos la fórmula : $\int \frac{dv}{v} = \ln v + c$.

$$(-) \int \frac{\text{sen } s \, (-)ds}{1 + \cos s} = -\ln (1 + \cos s) + c \ .$$

17. $\displaystyle\int \frac{\sec^2 x \, dx}{1 + \text{tg } x} =$

$v = 1 + \text{tg } x$ | El diferencial esta completo,

$dv = \sec^2 x \, dx$ | se procede a integrar.

$$\int \frac{\sec^2 x \, dx}{1 + \text{tg } x} = \ln(1 + \text{tg } x) + c \ .$$

18. $\int x \cos x^2\, dx = \frac{1}{2} \text{sen}\, x^2 + c$.

$\int \cos x^2 .\, x\;\, dx =$

$\left.\begin{array}{l} v = x^2 \\ dv = 2x\, dx \end{array}\right\}$ Falta (2) para completar el diferencial. Se aplica: $\int \cos v\;\, dv = \text{sen}\, v + c$.

$(2) \int \cos x^2 .(2)x\, dx = \frac{1}{2} \text{sen}\, x^2 + c$.

19. $\int (x + \text{sen}\, 2x)\, dx = 1/2\, (x^2 - \cos 2x) + c$.

$\int x\, dx + \int \text{sen}\, 2x\, dx =$

$\{v = 2x\;\; ;\;\, dv = = 2\, dx\}$

$\int x\, dx + \frac{1}{2} \int \text{sen}\, 2x .(2)\, dx = \frac{x^{1+1}}{1+1} + \frac{1}{2}\left(- \cos 2x\right) =$

$\frac{x^2}{2} - \frac{\cos 2x}{2} = \frac{1}{2}\left(x^2 - \cos 2x\right) + c$.

20. $\int \frac{\text{sen}\, x\, dx}{\sqrt{4 - \cos x}} = 2\sqrt{4 - \cos x} + c$.

$\int \frac{\text{sen}\, x\, dx}{(4 - \cos x)^{1/2}} = 2\sqrt{4 - \cos x} + c$.

$\int (4 - \cos x\,)^{-1/2}.\, \text{sen}\, x\, dx =$

$\left.\begin{array}{l} v = (4 - \cos x\,) \\ dv = -(- \text{sen}\, x)\, dx = \text{sen}\, x\, dx \end{array}\right\}$ El diferencial esta completo, se procede a integrar.

$$\int (4 - \cos x)^{-1/2} \cdot \operatorname{sen} x \, dx = \frac{(4 - \cos x)^{-1/2 + 1}}{-1/2 + 1} =$$

$$\frac{(4 - \cos x)^{1/2}}{1/2} = 2(4 - \cos x)^{1/2} = 2\sqrt{4 - \cos x} + c .$$

21. $$\int \frac{(1 + \cos x)\, dx}{x + \operatorname{sen} x} = \ln (x + \operatorname{sen} x) + c .$$

$$\left\{\begin{array}{l} v = x + \operatorname{sen} x \\ dv = (1 + \cos x)\, dx \end{array}\right\} \text{El diferencial esta completo, Aplicamos: } \int \frac{dv}{v} = \ln v + c .$$

$$\int \frac{(1 + \cos x)\, dx}{x + \operatorname{sen} x} = \ln (x + \operatorname{sen} x) + c .$$

22. $$\int \frac{\sec^2\theta \, d\theta}{\sqrt{1 + 2\operatorname{tg}\theta}} .$$

$$\int \frac{\sec^2\theta \, d\theta}{(1 + 2\operatorname{tg}\theta)^{1/2}} .$$

$$\int (1 + 2\operatorname{tg}\theta)^{-1/2} \cdot \sec^2\theta \, d\theta .$$

$$\left.\begin{array}{l} v = (1 + 2\operatorname{tg}\theta) \\ dv = 2\sec^2\theta \, d\theta \end{array}\right\} \text{Falta (2) para completar el diferencial.}$$

$$(1/2) \int (1 + 2\operatorname{tg}\theta)^{-1/2} . (2) \sec^2\theta \, d\theta .$$

$$\frac{1}{2}\left\{\frac{(1 + 2\operatorname{tg}\theta)^{-1/2+1}}{-1/2 + 1}\right\} = \frac{(1 + 2\operatorname{tg}\theta)^{1/2}}{2(1/2)} = \frac{(1 + 2\operatorname{tg}\theta)^{1/2}}{1} =$$

$$\sqrt{(1 + 2\operatorname{tg}\theta)} + c .$$

23. $\int \operatorname{sen} \frac{2x}{3}\, dx$

$\left.\begin{array}{l} v = \frac{2x}{3}. \\ dv = 2/3\ dx \end{array}\right\}$ Falta (2/3) para completar el diferencial. Se aplica : $\int \operatorname{sen} v\, dv = -\cos v + c$.

$\left(\frac{3}{2}\right)\left\{\int \operatorname{sen} \frac{2x}{3} \left(\frac{2}{3}\right) dx\right\} = \frac{3}{2}\left\{-\cos\left(\frac{2x}{3}\right)\right\} = -\frac{3}{2}\left\{\cos \frac{2x}{3}\right\} + c$

24. $\int \cos (b + ax)\, dx$

$\left.\begin{array}{l} v = (b + ax) \\ dv = a\, dx \end{array}\right\}$ Falta (a) para completar el diferencial. Se aplica : $\int \cos v\, dv = \operatorname{sen} v + c$.

$.\frac{1}{a}. \int \cos (b + ax). (a)\, dx = \frac{1}{a}. \operatorname{sen}(b + ax) = \frac{\operatorname{sen}(b + ax)}{a} + c$.

25. $\int \csc^2 (a - bx)\, dx = \int \{\csc (a - bx)\}^2 .dx$

$\{v = a - bx \ ; \ dv = - b\, dx\}$ Falta(-b) para completar el diferencial. Se aplica: $\int \csc^2 v\, dv = - \cot v + c$.

$\left(\frac{-1}{b}\right) \int \{\csc^2 (a - bx)\} .(- b)\, dx = \frac{-1}{b} \left[- \cot (a - bx)\right] =$

$\frac{\cot (a - bx)}{b} + c$.

26. $\int \sec \frac{\theta}{2} \operatorname{tg} \frac{\theta}{2}\, d\theta$

$\left.\begin{array}{l} v = \theta/2 \ . \\ dv = 1/2 \ . \ d\theta \end{array}\right\}$ Falta (1/2) para completar el diferencial, $\int \sec v \operatorname{tg} v\, dv = \sec v + c$.

$(2)\int \sec\frac{\theta}{2}\,tg\frac{\theta}{2}\,(1/2)d\theta = 2\sec\frac{\theta}{2} + c.$

27. $\int \csc\frac{a\varphi}{b}\cot\frac{a\varphi}{b}\,d\varphi$

$v = \frac{a\varphi}{b}$

$dv = \frac{a}{b}\,.\,d\varphi$

Falta (a/b) para completar el diferencial,
Se aplica: $\int \csc v \cot v\,dv = -\csc v + c.$

$\frac{b}{a}\int \csc\frac{a\varphi}{b}\cot\frac{a\varphi}{b}\,.\left(\frac{a}{b}\right)d\varphi = \frac{b}{a}.\left\{-\csc\frac{a\varphi}{b}\right\} =$

$\frac{-b}{a}\csc\frac{a\varphi}{b} + c.$

28. $\int e^x \cot e^x\,dx$

$v = e^x$

$dv = e^x\,dx$

El diferencial esta completo,
se procede a integrar.

$\int \cot e^x\,.\,e^x\,dx = \ln\{sen\,(e^x)\} + c.$

29. $\int \sec^2 2ax\,dx =$

$v = 2ax$

$dv = 2a\,dx$

Falta (2a) para completar el diferencial.

$(1/2a)\int \sec^2 2ax.(2a)\,dx = \frac{1}{2a}.tg\,2ax = \frac{tg\,2a}{2a} + c.$

30. $\int tg\frac{x}{3}\,dx$

$v = x/3$. Falta (1/3) para completar el diferencial.
$dv = 1/3\ dx$ Se aplica: $\int tg\ v\ dv = -\ln \cos v + c = \ln \sec v + c$.
$dv = \frac{1}{3}\ dx$ luego se procede a integrar.

$$(3)\int tg\frac{x}{3}(1/3)\,dx = 3\{-\ln\cos\frac{x}{3}\} = 3\ln\sec\frac{x}{3} + c\ .$$

31. $\int \frac{dt}{tg\ 5t}$.

$\int \cot 5t\ dt$.

$v = 5t$ Falta (5) para completar el diferencial
$dv = 5\ dt$ luego se procede a integrar.

$$(1/5)\int \cot 5t\ dt = \frac{1}{5}\ln sen\ 5t = \frac{\ln 5t}{5} + c\ .$$

32. $\int \frac{d\theta}{sen^2 4\theta}$.

Por trigonometria: $1/sen^2 4\theta = csc^2 4\theta$.

$$\int \frac{d\theta}{sen^2 4\theta} = \int csc^2 4\theta\ d\theta.$$

$v = 4\theta$ Falta (4) para completar el diferencial,
$dv = 4\ d\theta$ luego se procede a integrar.

$$\int csc^2 4\theta\ d\theta = \frac{1}{4}\{-\cot 4\theta\} = \frac{-\cot 4\theta}{4} + c\ .$$

33. $\int \frac{dy}{\cot 7y}$.

$\int tg\ 7y\ dy =$

$\left.\begin{array}{l} v = 7y \\ dv = 7\ dy \end{array}\right\}$ Falta (4) para completar el diferencial, luego se procede a integrar.
Se aplica: $\int tg\ v\ dv = -\ln\cos v + c = \ln\sec v + c$.

$(1/7)\int tg\ 7y\ .(7)\ dy = \frac{1}{7}\{-\ln\cos 7y\} = -\frac{\ln\cos 7y}{7} =$

$\frac{1}{7}\ln\cos 7y + c$.

34. $\int \frac{\text{sen}\sqrt{x}\ dx}{\sqrt{x}}$

$\left.\begin{array}{l} v = \sqrt{x} \\ dv = \frac{1}{2\sqrt{x}}.\ dx \end{array}\right\}$ Falta $\frac{1}{2}$ para completar el diferencial, luego se procede a integrar.

2 $(2)\int \text{sen}\sqrt{x}\ dx\ .\frac{1}{2}.\frac{1}{\sqrt{x}}.\ dx = 2(-\cos\sqrt{x}) = -2\cos\sqrt{x} + c$.

35. $\int \frac{dt}{\text{sen}^2 3t}$.

$\int \csc^2 3t\ dt$

$\left.\begin{array}{l} v = 3t \\ dv = 3\ dt \end{array}\right\}$ Falta (3) para completar el diferencial. Se aplica: $\int \csc^2 v\ dv = -\cot v + c$.

$(1/3)\int \csc^2 3t\ .(3)\ dt = \frac{1}{3}(-\cot 3t) = \frac{-\cot 3t}{3} + c$.

36. $\int \dfrac{d\varphi}{\cos 4\varphi}$.

Por Trigonometría: $1/\cos 4\varphi = \sec 4\varphi$.

$\int \sec 4\varphi \; d\varphi$.

$\left.\begin{matrix} v = 4\varphi \\ dv = 4\, d\varphi \end{matrix}\right\}$ Falta (4) para completar el diferencial, se aplica: $\int \sec v\, dv = \ln(\sec v + tg\, v) + c$.

$(1/4)\int \sec 4\varphi \;.(4)\, d\varphi = 1/4\,\{\ln(\sec 4\varphi + tg\, 4\varphi)\} + c$.

37. $\int \dfrac{a\, dx}{\cos^2 bx}$.

Por trigonometría: $1/\cos^2 bx = \sec^2 bx$.

$a\int \sec^2 bx\, dx =$

$\left.\begin{matrix} v = bx \\ dv = b\, dx \end{matrix}\right\}$ Falta (4) para completar el diferencial, $\int \sec^2 v\, dv = tg\, v + c$.

$\dfrac{a}{b}\int \sec^2 bx\, .(b)\, dx = \dfrac{a}{b}\, tg\, bx = \dfrac{a\, tg\, bx}{b} + c$.

38. $\int \left(\sec 2\theta - \csc \dfrac{\theta}{2}\right) d\theta$.

$\int \sec 2\theta\, d\theta - \int \csc \dfrac{\theta}{2}\, d\theta$.

$\left\{\begin{matrix} v = 2\theta \\ dv = 2\, d\theta \end{matrix}\right\}$ $\qquad$ $\left\{\begin{matrix} v = \theta/2 \\ dv = 1/2\, d\theta \end{matrix}\right\}$

$(1/2)\int \sec 2\theta\ .(2)d\theta - (2)\int \csc \frac{\theta}{2}\ .\frac{1}{2}.)d\theta$.

$\frac{1}{2}\{\ln(\sec 2\theta + \operatorname{tg} 2\theta)\} - 2\{\ln \csc\frac{\theta}{2} - \cot\frac{\theta}{2}\} + c$.

39. $\int (\operatorname{tg}\varphi + \sec\varphi)^2\, d\varphi$

$\int \{\operatorname{tg}^2\varphi + 2\operatorname{tg}\varphi\ \sec\varphi + \sec^2\varphi\}\, d\varphi$

Por Trigonometría: $\operatorname{tg}^2\varphi = \sec^2\varphi - 1$. Sustituyendo en la integral .

$\int \{\sec^2\varphi - 1 + 2\operatorname{tg}\varphi\ \sec\varphi + \sec^2\varphi\}\, d\varphi$.

$2\int \sec^2\varphi\ d\varphi - \int d\varphi + 2\int \operatorname{tg}\varphi\ \sec\varphi\ \}\, d\varphi$.

$2\operatorname{tg}\varphi - \varphi + 2\sec\varphi + c$.

40. $\int (\operatorname{tg} 4s - \cot\frac{s}{4})\, ds$.

$\frac{1}{4}\int \operatorname{tg} 4s\ .(4)\, ds - (4)\int \cot\frac{s}{4}.\frac{1}{4}.ds = \frac{1}{4}\ln\{\sec 4s\} - 4\ln \operatorname{sen}\frac{s}{4} =$

$\{\frac{\ln \sec 4s}{4}\} - 4\ln \operatorname{sen}\frac{s}{4} + c$.

41. $\int (\cot x - 1)^2\, dx$

$\int (\cot^2 x - 2\cot x + 1)\, dx$
Pero: $1 + \cot^2 x = \csc^2 x$, reemplazando en la integral.

$\int (\csc^2 x - 2\cot x)\, dx$

$\int \csc^2 x\, dx - 2\int \cot x\, dx = -\cot x - 2\ln(\text{sen } x) = -[\cot x + 2\ln(\text{sen } x)]$

$-\{\cot x + \ln(\text{sen } x)^2\} = -\{\cot x + \ln(\text{sen}^2 x)\} + c$.

42. $\int (\sec t - 1)^2\, dt$ **.**

$\int (\sec^2 t - 2\sec t + 1)\, dt$.

$\int \sec^2 t\, dt - 2\int \sec t\, dt + \int dt$.

$\text{tg } t - 2\ln(\sec t + \text{tg } t) + t + c$.

43. $\int (1 - \csc y)^2\, dy$ **.**

$\int (1 - 2\,.\,1\,.\,\csc y + \csc^2 y)\, dy = \int (1 - 2\csc y + \csc^2 y)\, dy$.

$\int dy - 2\int \csc y\, dy + \int \csc^2 y\, dy$.

$y - 2\ln(\csc y - \cot y) - \cot y + c$.

44. $\int \dfrac{dx}{1 - \cos x}$ **.**

Racionalizando: $\dfrac{1}{(1 - \cos x)}$

$\left(\dfrac{1}{1 - \cos x}\right)\left(\dfrac{1 + \cos x}{1 + \cos x}\right) = \dfrac{1 + \cos x}{1^2 - \cos^2 x} = \dfrac{1 + \cos x}{\text{sen}^2 x} =$

$\dfrac{1}{\text{sen}^2 x} + \dfrac{\cos x}{\text{sen}^2 x} = \csc^2 x + \dfrac{\cos x}{\text{sen}^2 x}$.

$\int \csc^2 x + \int \dfrac{\cos x\, dx}{\text{sen}^2 x} = \int \csc^2 x + \int (\text{sen } x)^{-2}\,.\,\cos x\, dx =$

$$- \cot x + \frac{(\text{sen } x)^{-2+1}}{-2+1} = - \cot x + \frac{(\text{sen } x)^{-1}}{-1} = - \cot x - (\text{sen } x)^{-1} =$$

$$- \cot x - \frac{1}{\text{sen } x} = - \cot x - \csc x = - (\cot x + \csc x) + c .$$

45. $\displaystyle \int \frac{dx}{1 - \text{sen } x}$.

Racionalizando:

$$\left(\frac{1}{1 - \text{sen } x}\right)\left(\frac{1 + \text{sen } x}{1 + \text{sen } x}\right) = \frac{1 + \text{sen } x}{1 - \text{sen}^2 x} = \frac{1 + \text{sen } x}{\cos^2 x} .$$

$$\int \frac{1 + \text{sen } x}{\cos^2 x} dx = \int \frac{1\ dx}{\cos^2 x} + \int \frac{\text{sen } x\ dx}{\cos^2 x} .$$

$$\int \sec^2 x\ dx + \int (\cos x)^{-2} . \text{ sen } x\ dx = \text{tg } x - \frac{(\cos x)^{-2+1}}{-2+1} =$$

$$\text{tg } x - \frac{(\cos x)^{-1}}{-1} = \text{tg } x + \frac{1}{\cos x} = \text{tg } x + \sec x + c .$$

46. $\displaystyle \int \frac{\text{sen } 2x\ dx}{3 + \cos 2x}$.

$$\left.\begin{array}{l} v = 3 + \cos 2x \\ dv = - 2 \text{ sen } 2x\ dx \end{array}\right\} \begin{array}{l} \text{Falta (-2) para completar el diferencial,} \\ \text{se aplica: } \displaystyle\int \frac{dv}{v} = \ln v + c . \end{array}$$

$$\left(\frac{-1}{2}\right) \int \frac{(-2) \text{ sen } 2x\ dx}{3 + \cos 2x} = - \frac{1}{2} \ln (3 + \cos 2x) + c .$$

47. $\displaystyle\int \frac{\cos t\, dt}{\sqrt{a + b\ \text{sen}\ t}}$.

$\displaystyle\int \frac{\cos t\, dt}{(a + b\ \text{sen}\ t)^{1/2}} = \int (a + b\ \text{sen}\ t)^{-1/2} .\cos t\ dt =$

$\left.\begin{array}{l} v = (a + b\ \text{sen}\ t) \\ dv = b \cos t\ dt \end{array}\right\}$ Falta (b) para completar el diferencial, Se aplica: $\displaystyle\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$.

$\displaystyle\frac{1}{b}.\int (a + b\ \text{sen}\ t)^{-1/2}.(b)\cos t\ dt = \frac{(a + b\ \text{sen}\ t)^{-1/2+1}}{(b)(-1/2 + 1)} = \frac{(a + b\ \text{sen}\ t)^{1/2}}{1/2\ (b)} =$

$\displaystyle\frac{\;\frac{(a + b\ \text{sen}\ t)^{1/2}}{1}\;}{\frac{b}{2}} = \frac{2\,(a + b\ \text{sen}\ t)^{1/2}}{b} = \frac{2\sqrt{(a + b\ \text{sen}\ t)}}{b} + c$.

48. $\displaystyle\int \frac{\csc\theta\ \cot\theta\ d\theta}{5 - 4\csc\theta}$

$\left.\begin{array}{l} v = 5 - 4\csc\theta \\ dv = -4\csc\theta\ \cot\theta\ d\theta \end{array}\right\}$ Falta (- 4) para completar el diferencial, Se aplica: $\displaystyle\int \frac{dv}{v} = \ln v + c$.

$\displaystyle\left(\frac{-1}{4}\right)\int \frac{(-4).\csc\theta\ \cot\theta\ d\theta}{5 - 4\csc\theta}$

$\displaystyle\left(\frac{-1}{4}\right)\ln(5 - 4\csc\theta) + c$.

49. $\displaystyle\int \frac{\csc^2 x\ dx}{\sqrt{3 - \cot x}}$.

$$\int \frac{\csc^2 x\, dx}{(3 - \cot x)^{1/2}} = \int (3 - \cot x)^{-1/2} . \csc^2 x\, dx$$

$$\left.\begin{aligned} v &= 3 - \cot x \\ dv &= \csc^2 x\, dx \end{aligned}\right\}$$ El diferencial esta completo.
Se aplica: $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$.

$$\frac{(3 - \cot x)^{-1/2+1}}{-1/2 + 1} = \frac{(3 - \cot x)^{1/2}}{1/2} = 2(3 - \cot x)^{1/2} =$$

$$2 \sqrt{(3 - \cot x)} + c .$$

50. $\int \frac{\sqrt{5 + 2tg\, x}}{\cos^2 x} dx$

$$\int \sqrt{5 + 2tg\, x} . \frac{1}{\cos^2 x} . dx = \int \sqrt{5 + 2tg\, x} . \sec^2 x\, dx$$

$$\int (5 + 2tg\, x)^{1/2} . \sec^2 x\, dx .$$

$$\left.\begin{aligned} v &= (5 + 2tg\, x) \\ dv &= 2 \sec^2 x\, dx \end{aligned}\right\}$$ Falta (2) para completar el diferencial,
Se aplica: $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$.

$$\left(\frac{1}{2}\right) \int (5 + 2tg\, x)^{1/2} .(2) \sec^2 x\, dx = .\frac{1}{2}. \frac{(5 + 2tg\, x)^{1/2+1}}{1/2 + 1} =$$

$$\frac{(5 + 2tg\, x)^{3/2}}{2(3/2)} = \frac{(5 + 2tg\, x)^{3/2}}{3} = \frac{\sqrt{(5 + 2tg\, x)^3}}{3} =$$

$$\frac{\sqrt{(5 + 2tg\, x)^2.(5 + 2tg\, x)}}{3} = \frac{(5 + 2tg\, x) \sqrt{(5 + 2tg\, x)}}{3} + c .$$

Problemas. Pagina 248 y 249

Verificar las siguientes Integraciones:

1. $\int \frac{dx}{x^2 + 9}$.

$\int \frac{dx}{x^2 + 3^2}$.

$$\left.\begin{array}{l} v = x \\ dv = dx \\ a = 3 \end{array}\right\} \begin{array}{l}\text{El diferencial esta completo, se aplica:} \\ \int \frac{dv}{v^2 + a^2} = \frac{1}{a} \text{ arc tg } \frac{v}{a} + c .\end{array}$$

$$\int \frac{dx}{x^2 + 3^2} = \frac{1}{3} .\text{arc tg } \frac{x}{3} + c .$$

2. $\int \frac{dx}{x^2 - 4}$.

$\int \frac{dx}{x^2 - 2^2}$.

$$\left.\begin{array}{l} v = x \\ dv = dx \\ a = 2 \end{array}\right\} \begin{array}{l}\text{El diferencial esta completo, se aplica:} \\ \int \frac{dv}{v^2 - a^2} = \frac{1}{2a} . \ln \left| \frac{v - a}{v + a} \right| + c .\end{array}$$

$$\int \frac{dx}{x^2 - 2^2} = \frac{1}{2(2)} . \ln \left| \frac{x - 2}{x + 2} \right| = \frac{1}{4} \ln \left| \frac{x - 2}{x + 2} \right| + c .$$

3. $\int \dfrac{dy}{\sqrt{25 - y^2}}$.

$$\left.\begin{array}{l} v = y \\ dv = dy \\ a = 5 \end{array}\right\} \begin{array}{l} \text{El diferencial esta completo. Se aplica:} \\ \int \dfrac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen } \dfrac{v}{a} + c\,. \end{array}$$

$$\int \frac{dy}{\sqrt{5^2 - y^2}} = \text{arc sen } \frac{y}{5} + c\,.$$

4. $\int \dfrac{ds}{\sqrt{s^2 - 16}}$.

$$\int \frac{ds}{\sqrt{s^2 - 4^2}}\,.$$

$$\left.\begin{array}{l} v = s \\ dv = ds \\ a = 4 \end{array}\right\} \begin{array}{l} \text{El diferencial esta completo.} \\ \text{Se aplica: } \int \dfrac{dv}{\sqrt{v^2 - a^2}} = \ln \{ v + \sqrt{v^2 - a^2} \} + c\,. \end{array}$$

$$\int \frac{ds}{\sqrt{s^2 - 4^2}} = \ln \{ s + \sqrt{s^2 - 16} \} + c\,.$$

5. $\int \dfrac{dx}{9x^2 - 4}$.

$$\int \frac{dv}{(3x)^2 - 2^2}.\ \left\{\begin{array}{l} v = 3x \\ dv = 3\ dx \\ a = 2 \end{array}\right\} \begin{array}{l} \text{Falta (3) para completar el diferencial} \\ \text{Se aplica: } \int \dfrac{dv}{v^2 - a^2} = \dfrac{1}{2a} . \ln \dfrac{v - a}{v + a} + c\,. \end{array}$$

$$\left(\frac{1}{3}\right) \int \frac{(3)\ dx}{(3x)^2 - 2^2} = \left(\frac{1}{3}\right) \left(\frac{1}{2(2)}\right) \ln\left(\frac{3x - 2}{3x + 2}\right) = \frac{1}{12} . \ln \left(\frac{3x - 2}{3x + 2}\right) + c\,.$$

6. $\int \dfrac{dx}{\sqrt{16 - 9x^2}}$.

$$\int \frac{dx}{\sqrt{4^2 - (3x)^2}}.$$

$$\left.\begin{array}{l} v = 3x \\ dv = 3\ dx \\ a = 4 \end{array}\right\} \begin{array}{l} \text{Falta (3) para completar el diferencial.} \\ \text{Se aplica: } \displaystyle\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen } \frac{v}{a} + c\,. \end{array}$$

$$\left(\frac{1}{3}\right) \int \frac{(3)\ dx}{\sqrt{4^2 - (3x)^2}} = \frac{1}{3}.\text{arc sen } \frac{3x}{4} + c\,.$$

7. $\displaystyle\int \frac{\mathbf{dx}}{\mathbf{9x^2 - 1}}$.

$$\int \frac{dx}{(3x)^2 - 1^2}\,.$$

$$\left.\begin{array}{l} v = 3x \\ dv = 3\ dx \\ a = 1 \end{array}\right\} \begin{array}{l} \text{Falta (3) para completar el diferencial. Se aplica:} \\ \displaystyle\int \frac{dv}{v^2 - a^2} = \frac{1}{2a}.\ \ln\left\{\frac{v - a}{v + a}\right\}. \end{array}$$

$$\int \frac{dx}{(3x)^2 - 1^2} = \frac{1}{3}.\frac{1}{1(2)}.\ \ln\left\{\frac{3x - 1}{3x + 1}\right\} = \frac{1}{6}\ \ln\left\{\frac{3x - 1}{3x + 1}\right\} + c\,.$$

8. $\displaystyle\int \frac{\mathbf{dt}}{\mathbf{4 - 9t^2}}$.

$$\int \frac{dt}{2^2 - (3t)^2}\,.$$

$$\left.\begin{array}{l} v = 3t \\ dv = 3\ dt \\ a = 2 \end{array}\right\} \begin{array}{l} \text{Falta (3) para completar el diferencial.} \\ \displaystyle\int \frac{dv}{v^2 - a^2} = \frac{1}{2a}.\ln\left\{\frac{v - a}{v + a}\right\} + c\,. \end{array}$$

$$\left(\frac{1}{3}\right)\int \frac{(3)\,dt}{2^2 - (3t)^2} = \frac{1}{3}\cdot\frac{1}{2(2)}\cdot \ln\left\{\frac{2+3t}{2-3t}\right\} = \frac{1}{12}\cdot\ln\left\{\frac{2+3t}{2-3t}\right\} + c\,.$$

9. $\displaystyle\int \frac{e^x\,dx}{1+e^{2x}}$

$$\int \frac{e^x\,dx}{1^2+(e^x)^2}.$$

$\left.\begin{array}{l} v = e^x \\ dv = e^x\,dx \\ a = 1 \end{array}\right\}$ El diferencial esta completo. Se aplica: $\displaystyle\int \frac{dv}{a^2+v^2} = \frac{1}{a}\,\text{arc tg}\,\frac{v}{a} + c\,.$

$$\int \frac{e^x\,dx}{1^2+(e^x)^2} = \frac{1}{1}.\text{arc tg}\left\{\frac{e^x}{1}\right\} = \text{arc tg}\, e^x + c\,.$$

10. $\displaystyle\int \frac{\cos\theta\;d\theta}{4 - \text{sen}^2\theta}$

$$\int \frac{\cos\theta\;d\theta}{2^2 - (\text{sen}\,\theta)^2}.$$

$\left\{\begin{array}{l} v = \text{sen}\;\theta \\ dv = \cos\theta\;d\theta \\ a = 2 \end{array}\right\}$ El diferencial esta completo, se procede a integrar. $\displaystyle\int \frac{dv}{a^2 - v^2} = \frac{1}{2a}.\ \ln\left|\frac{a+v}{a-v}\right| + c\,.$

$$\int \frac{\cos\theta\;d\theta}{2^2-(\text{sen}\,\theta)^2} = \left(\frac{1}{2(2)}\right)\ \ln\left(\frac{2+\text{sen}\;\theta}{2-\text{sen}\;\theta}\right) = \frac{1}{4}\ \ln\left(\frac{2+\text{sen}\;\theta}{2-\text{sen}\;\theta}\right) + c\,.$$

11. $\displaystyle\int \frac{b\;dx}{a^2x^2 - c^2}.$

$$\int \frac{b\;dx}{(ax)^2 - c^2}.$$

$\left.\begin{array}{l} v = ax \\ dv = a\,dx \\ a = c \end{array}\right\}$ Falta (a) para completar el diferencial. $\displaystyle\int \frac{dv}{v^2 - a^2} = \frac{1}{2a}\ \ln\left(\frac{v-a}{v+a}\right) + c\,.$

$$\left(\frac{1}{a}\right)(b)\int \frac{(a)\,dx}{(ax)^2 - c^2} = \frac{b}{a}\cdot\frac{1}{2(c)}\cdot \ln\left(\frac{ax - c}{ax + c}\right) = \frac{b}{2ac}\cdot \ln\left(\frac{ax - c}{ax + c}\right) + c\,.$$

12. $\displaystyle\int \frac{5x\,dx}{\sqrt{1 - x^4}}$.

$$\int \frac{5x\,dx}{\sqrt{1^2 - (x^2)^2}}.$$

$v = x^2$
$dv = 2x\,dx$
$a = 1$

Falta (2) para completar el diferencial. Se aplica: $\displaystyle\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen}\left(\frac{v}{a}\right) + c$.

$$\frac{(5)}{2}\int \frac{(2)x\,dx}{\sqrt{1^2 - (x^2)^2}} = \frac{5}{2}.\text{arc sen}\ \frac{x}{1} = \left(\frac{5}{2}\right)\text{arc}\left(\text{sen } x\right) + c$$

13. $\displaystyle\int \frac{ax\,dx}{x^4 + b^4}$.

$$\int \frac{ax\,dx}{(x^2)^2 + (b^2)^2}.$$

$v = x^2$
$dv = 2x\,dx$
$a = b^2$

Falta (2) para completar el diferencial. Se aplica: $\displaystyle\int \frac{dv}{v^2 + a^2} = \frac{1}{a}\text{arc tg}\left(\frac{v}{a}\right) + c$.

$$\left(\frac{a}{2}\right)\int \frac{(2)\,ax\,dx}{(x^2)^2 + (b^2)^2} = \frac{a}{2}\cdot\frac{1}{b^2}\cdot \text{arc tg}\left(\frac{x^2}{b^2}\right) = \frac{a}{2b^2}\,\text{arc tg}\left(\frac{x^2}{b^2}\right) + c$$

14. $\displaystyle\int \frac{dt}{(t - 2)^2 + 9}$.

$$\int \frac{dt}{(t-2)^2+3^2} =$$

$\left.\begin{array}{l} v = t - 2 \\ dv = dt \\ a = 3 \end{array}\right\}$ El diferencial esta completo, se aplica: $\int \frac{dv}{v^2+a^2} = \frac{1}{a}.\ \text{arc tg}\ \frac{v}{a} + c$.

$$\frac{1}{3}.\ \text{arc tg}\left(\frac{t-2}{3}\right) + c\ .$$

15. $\int \frac{dy}{\sqrt{1+a^2y^2}}$.

$\left\{\begin{array}{l} v = ay \\ dv = a\,dy \\ a = 1 \end{array}\right\}$ Falta (a) para completar el diferencial, se aplica: $\int \frac{dv}{\sqrt{a^2+v^2}} = \ln\{v+\sqrt{a^2+v^2}\} + c$.

$$\frac{1}{a}\int \frac{(a)\,dy}{\sqrt{1+(ay)^2}} = \frac{1}{a}.\int \frac{(a)\,dy}{\sqrt{(ay)^2+1^2}} = \frac{1}{a}\ln\{ay+\sqrt{1+a^2y^2}\} + c\ .$$

16. $\int \frac{du}{\sqrt{4-(u+3)^2}}$.

$$\int \frac{du}{\sqrt{2^2-(u+3)^2}}\ .$$

$\left.\begin{array}{l} v = u + 3 \\ dv = du \\ a = 2 \end{array}\right\}$ El diferencial esta completo, se procede a integrar. Se aplica: $\int \frac{dv}{\sqrt{a^2-v^2}} = \text{arc sen}\ \frac{v}{a} + c$.

$$\int \frac{du}{\sqrt{2^2-(u+3)^2}} = \text{arc sen}\ \frac{u+3}{2} + c\ .$$

17. $\int \frac{dx}{\sqrt{9 - 16x^2}}$.

$\int \frac{dx}{\sqrt{3^2 - (4x)^2}}$.

$\left.\begin{array}{l} v = 9 - 16x^2 \\ dv = 4\ dx \\ a = 3 \end{array}\right\}$ Falta (4) para completar el diferencial, se aplica: $\int \frac{dx}{\sqrt{a^2 - v^2}} = \text{arc sen } \frac{v}{a} + c$.

$\left(\frac{1}{4}\right) \int \frac{(4)dx}{\sqrt{3^2 - (4x)^2}} = \frac{1}{4}. \text{ arc sen } \frac{4x}{3} + c$.

18. $\int \frac{dy}{\sqrt{9y^2 + 4}}$.

$\int \frac{dy}{\sqrt{(3y)^2 + 2^2}}$.

$\left\{\begin{array}{l} v = 3y \\ dv = 3\ dy \\ a = 2 \end{array}\right\}$ Falta (3)para completar el diferencial.
Se aplica: $\int \frac{dv}{\sqrt{v^2 + a^2}} = \ln \{v + \sqrt{v^2 + a^2}\} + c.$

$\left(\frac{1}{3}\right) \int \frac{(3)\ dy}{\sqrt{(3y)^2 + 2^2}} = \frac{1}{3}. \ln \{3y + \sqrt{(3y)^2 + 2^2}\} =$

$\frac{\ln \{3y + \sqrt{9y^2 + 4}\}}{3} + c$

19. $\int \frac{dt}{4t^2 + 25}$.

$\int \frac{dt}{(2t)^2 + 5^2}$.

$$\left\{\begin{array}{l} v = 2t \\ dv = 2\ dt \\ a = 5 \end{array}\right\} \text{Falta (2) para completar el diferencial, se aplica: } \int \frac{dv}{\sqrt{v^2 + a^2}} = \ln\{v + \sqrt{v^2 + a^2}\} + c.$$

$$\left(\frac{1}{2}\right) \int \frac{(2)dt}{(2t)^2 + 5^2} = \frac{1}{5}.\ \text{arc tg}\ \frac{2t}{5} + c\ .$$

20. $\displaystyle\int \frac{dx}{25x^2 - 4}$.

$$\int \frac{dx}{(5x)^2 - 2^2}\ .$$

$$\left.\begin{array}{l} v = 5x \\ dv = 5\ dx \\ a = 2 \end{array}\right\} \text{Falta (5) para completar el diferencial, se aplica: } \int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln\left(\frac{v - a}{v + a}\right) + c\ .$$

$$\left(\frac{1}{5}\right) \int \frac{(5)\ dx}{(5x)^2 - 2^2} = \frac{1}{5}\left\{\frac{1}{2(2)} \ln\left(\frac{5x - 2}{5x + 2}\right)\right\} = \frac{1}{20} \ln\left(\frac{5x - 2}{5x + 2}\right) + c$$

21. $\displaystyle\int \frac{7\ dx}{3 + 7x^2}$.

$$\int \frac{7\ dx}{(\sqrt{3})^2 + (\sqrt{7}.x)^2}\ .$$

$$\left.\begin{array}{l} v = \sqrt{7}.\ x \\ dv = \sqrt{7}\ dx \\ a = \sqrt{3} \end{array}\right\} \text{Falta (7) para completar el diferencial, se aplica: } \int \frac{dv}{a^2 + v^2} = \frac{1}{a}\ \text{arc tg}\ \frac{v}{a} + c\ .$$

$$\left(\frac{1}{\sqrt{7}}\right) \int \frac{\sqrt{7}\ dx}{(\sqrt{3})^2 + (\sqrt{7}.x)^2} = \frac{1}{\sqrt{7}}\left\{\frac{1}{\sqrt{3}}\ \text{arc tg}\ \frac{\sqrt{7}.x}{\sqrt{3}}\right\} =$$

$$\frac{1}{\sqrt{21}} \text{ arc tg } \frac{\sqrt{7}.x}{\sqrt{3}} + c .$$

$$\frac{\sqrt{21}}{\sqrt{21}.\sqrt{21}} . \text{ arc tg}\left(\frac{\sqrt{7}.\ \sqrt{3}.x}{\sqrt{3}.\ \sqrt{3}}\right) = \frac{\sqrt{21}}{21} \text{ arc tg } \left(\frac{\sqrt{21}.\ x}{3}\right) + c .$$

22. $\int \frac{3\ dy}{9y^2 - 16}$.

$$\int \frac{3\ dy}{(3y)^2 - 4^2} .$$

$\left.\begin{array}{l} v = 3y \\ dv = 3\ dy \\ a = 4 \end{array}\right\}$ El diferencial esta completo, se procede a integrar. Se aplica: $\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} . \ln\left|\frac{v - a}{v + a}\right|$

$$\int \frac{3\ dy}{(3y)^2 - 4^2} = \frac{1}{2(4)} . \ln\left|\frac{3y - 4}{3y + 4}\right| = \frac{1}{8} \ln \left|\frac{3y - 4}{3y + 4}\right| = \ln \left|\frac{3y - 4}{3y + 4}\right|^{1/8} + c .$$

23. $\int \frac{ds}{\sqrt{4s^2 + 5}}$.

$$\int \frac{ds}{\sqrt{(2s)^2 + (\sqrt{5})^2}}.$$

$\left.\begin{array}{l} v = 2s \\ dv = 2\ ds \\ a = \sqrt{5} \end{array}\right\}$ Falta (2) para conmpletar el diferencial, se aplica: $\int \frac{dv}{\sqrt{v^2 + a^2}} = \ln \{v + \sqrt{v^2 + a^2}\} + c$.

$$\left(\frac{1}{2}\right) \int \frac{(2)ds}{\sqrt{(2s)^2 + (\sqrt{5})^2}} = \frac{1}{2} \{\ln [2s + (\sqrt{4s^2 + 5})]\} + c .$$

24. $\int \frac{t\,dt}{\sqrt{t^4 - 4}}$.

$\int \frac{t\,dt}{\sqrt{(t^2)^2 - (2)^2}}$.

$$\left.\begin{array}{l} v = t^2 \\ dv = 2t\,dt \\ a = 2 \end{array}\right\} \begin{array}{l}\text{Falta (2) para completar el diferencial, se aplica:} \\ \int \frac{dv}{\sqrt{v^2 - a^2}} = \ln\{v + \sqrt{v^2 - a^2}\} + c .\end{array}$$

$$\left(\frac{1}{2}\right)\int \frac{(2)t\,dt}{\sqrt{(t^2)^2 - (2)^2}} = \frac{1}{2}\{\ln[t^2 + (\sqrt{t^4 - 4})]\} + c .$$

25. $\int \frac{x\,dx}{\sqrt{5x^2 + 3}}$.

$\int (5x^2 + 3)^{-1/2}.\ x\,dx$.

$$\left.\begin{array}{l} v = 5x^2 + 3 \\ dv = 10x\,dx \\ n = -1/2 \end{array}\right\} \begin{array}{l}\text{Falta (10) para completar el diferencial, se aplica:} \\ \int v^n\,dv = v^{n+1} + c .\end{array}$$

$$\frac{1}{10}.\int (5x^2 + 3)^{-1/2}.(10)\,x\,dx = \frac{1}{10}.\frac{(5x^2 + 3)^{-1/2+1}}{-1/2+1} =$$

$$\frac{(5x^2 + 3)^{1/2}}{10(1/2)} = \frac{\sqrt{5x^2 + 3}}{5} + c .$$

26. $\int \frac{2e^x\,dx}{\sqrt{1 - e^{2x}}}$.

$\int \dfrac{2e^x\,dx}{\sqrt{1^2 - (e^x)^2}}$.

$\left.\begin{array}{l} v = e^x \\ dv = e^x\,dx \\ a = 1 \end{array}\right\}$ El diferencial esta completo, se procede a integrar. Se aplica: $\int \dfrac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen}\,\dfrac{v}{a} + c$.

$2\int \dfrac{e^x\,dx}{\sqrt{1^2 - (e^x)^2}} = 2\,\text{arc sen}\,\dfrac{e^x}{1} = 2\,\text{arc sen}\;e^x + c$.

27. $\int \dfrac{\mathbf{6t\,dt}}{\mathbf{8 - 3t^2}}$.

$\left.\begin{array}{l} v = 8 - 3t^2 \\ dv = -\,6t\,dt \end{array}\right\}$ Falta el signo (-) para completar el diferencial, se usa la fórmula: $\int \dfrac{dv}{v} = \ln v + c$.

$(-)\int \dfrac{(-)\,6t\,dt}{8 - 3t^2} = -\ln(8 - 3t^2) + c$.

28. $\int \dfrac{\mathbf{sen\,\theta}}{\sqrt{\mathbf{4 + cos^2\,\theta}}}$.

$\int \dfrac{\text{sen}\,\theta\;d\theta}{\sqrt{2^2 + (\cos\theta)^2}}$.

$\left.\begin{array}{l} v = \cos\theta \\ dv = -\,\text{sen}\,\theta\;d\theta \\ a = 2 \end{array}\right\}$ Falta el signo (-) para completar el diferencial.

Se aplica: $\int \dfrac{dv}{\sqrt{a^2 + v^2}} = \ln\{v + \sqrt{a^2 + v^2}\} + c$.

$$(-)\int \frac{(-)\text{sen}\ \theta\ d\theta}{\sqrt{2^2 + (\cos\theta)^2}} = -\ln\{\cos\theta + \sqrt{4 + \cos^2\theta}\} + c.$$

29. $\displaystyle\int \frac{dx}{m^2 + (x+n)^2}$.

$v = x + n$
$dv = dx$

El diferencial esta completo, se procede a integrar.
Se aplica: $\displaystyle\int \frac{dv}{a^2 + v^2} = \frac{1}{a}.\ \text{arc tg}\left(\frac{v}{a}\right) + c$.

$$\int \frac{dx}{m^2 + (x+n)^2} = \frac{1}{m}.\ \text{arc tg}\ \frac{x+n}{m} + c$$

30. $\displaystyle\int \frac{du}{4 - (2u-1)^2}$.

$$\int \frac{du}{2^2 - (2u-1)^2}.$$

$v = 2u - 1$
$dv = 2\ du$
$a = 2$

Falta el (2) para completar el diferencial, se aplica:
$\displaystyle\int \frac{dv}{a^2 - v^2} = \frac{1}{2a}.\ \ln\left\{\frac{a+v}{a-v}\right\} + c$.

$$\left(\frac{1}{2}\right)\int \frac{(2)\ du}{2^2 - (2u-1)^2} = \frac{1}{2}.\ \frac{1}{2.2}.\ \ln\left\{\frac{2 + (2u-1)}{2 - (2u-1)}\right\} =$$

$$\frac{1}{8}.\ \ln\left\{\frac{2 + 2u - 1}{2 - 2u + 1}\right\} = \frac{1}{8}.\ \ln\left\{\frac{1 + 2u}{3 - 2u}\right\} + c.$$

31. $\displaystyle\int \frac{7x^2\ dx}{5 - x^6}$.

Haciendo cuadrado perfecto al # 5 ,y luego le extraemos la raiz cuadrada y lo elevamos al cuadrado:

$$\int \frac{7x^2\,dx}{(\sqrt{5})^2 - (x^3)^6}.$$

$v = x^3$
$dv = 3x^2\,dx$
$a = \sqrt{5}$

Falta (3) para completar el diferencial, el (7) se coloca fuera de la integral. Se aplica:

$$\int \frac{dv}{a^2 - v^2} = \frac{1}{2a} . \ln\left(\frac{a+v}{a-v}\right) + c .$$

$$\left(7.\frac{1}{3}\right)\int \frac{(3)x^2\,dx}{(\sqrt{5})^2 - (x^3)^6} = \frac{7}{3}.\frac{1}{2.\sqrt{5}}. \ln\left(\frac{\sqrt{5}+x^3}{\sqrt{5}-x^3}\right) = \frac{7}{6\sqrt{5}}. \ln\left(\frac{\sqrt{5}+x^3}{\sqrt{5}-x^3}\right) + c$$

$$\frac{7 . \sqrt{5}}{6\sqrt{5}.\sqrt{5}}. \ln\left(\frac{\sqrt{5}+x^3}{\sqrt{5}-x^3}\right) = \left(\frac{7 . \sqrt{5}}{6 . 5}\right). \ln\left(\frac{\sqrt{5}+x^3}{\sqrt{5}-x^3}\right) =$$

$$\frac{7 . \sqrt{5}}{30}. \ln\left(\frac{\sqrt{5}+x^3}{\sqrt{5}-x^3}\right) + c .$$

Problemas. Pagina 250 , 251 y 252.

Verificar las siguientes Integraciones:

1. $$\int \frac{dx}{x^2 + 4x + 3}.$$

Factorizar el denominador y hacerlo trinomio cuadrado perfecto:
Primero dividimos para (2) al coeficiente del 2$^{\underline{do}}$ término , y luego al resultado lo elevamos al cuadrado. $4/2 = 2$; $2^2 = 4$. Luego: sumamos y restamos "4" a : $x^2 + 4x + 3$.

$x^2 + 4x + 4 - 4 + 3 = x^2 + 4x + 4 - 1$.

$\underline{x^2 + 4x + 4}$, es un trinomio cuadrado perfecto: $(x + 2)^2$.

Tendremos:

$x^2 + 4x + 4 - 1 = (x + 2)^2 - 1 = (x + 2)^2 - 1^2$.

Sustituyendo este ultimo resultado en la integral; esta estará lista para desarrollarse, se usa la fórmula:

$$\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} . \ln \frac{v - a}{v + a} + c .$$

$$\int \frac{dx}{x^2 + 4x + 3} = \int \frac{dx}{(x + 2)^2 - 1^2} .$$

$$\left.\begin{array}{l} v = (x + 2) \\ dv = dx \\ a = 1 \end{array}\right\} \text{El diferencial esta completo.}$$

$$\int \frac{dx}{(x + 2)^2 - 1^2} = \frac{1}{2.1} . \ln \left(\frac{x + 2 - 1}{x + 2 + 1}\right) = \frac{1}{2} \ln \left(\frac{x + 1}{x + 3}\right) + c .$$

Nota.- Tambien habra casos en que se completa cuadrados a la cantidad sub-radical.
Este sera el arquetipo, en que se regiran los demas problemas.

2. $$\int \frac{dx}{2x - x^2 - 10} .$$

$$- x^2 + 2x - 10 = - (x^2 - 2x + 10) \ . \ \frac{2}{2} = 1 \ ; \quad 1^2 = 1$$

$$- (x^2 - 2x + 1 - 1 + 10) = - [(x - 1)^2 + 9] = - [(x - 1)^2 + 3^2]$$

$$\int \frac{dx}{-[(x-1)^2+3^2]} = -\int \frac{dx}{[(x-1)^2+3^2]}.$$

$v = x - 1$; $dv = dx$; $a = 3$ — El diferencial esta completo, se procede a integrar. Se emplea la fórmula: $\int \frac{dv}{v^2+a^2} = \frac{1}{a}.\text{arc tg}\,\frac{v}{a} + c$.

$$-\int \frac{dx}{[(x-1)^2+3^2]} = -\frac{1}{3}\,\text{arc tg}\,\frac{x-1}{3} + c.$$

3. $$\int \frac{3\,dx}{x^2-8x+25}.$$

$8/2 = 4$; $4^2 = 16$

$$x^2 - 8x + 16 - 16 + 25 = x^2 - 8x + 16 + 9 = [(x-4)^2+3^2]$$

$$\int \frac{3\,dx}{[(x-4)^2+3^2]}.$$

$v = x - 4$; $dv = dx$; $a = 3$ — El diferencial esta completo, se aplica: $\int \frac{dv}{v^2+a^2} = \frac{1}{a}\,\text{arc tg}\,\frac{v}{a} + c$.

$$(3)\int \frac{3\,dx}{[(x-4)^2+3^2]} = \frac{\cancel{3}}{\cancel{3}}.\frac{1}{}.\ \text{arc tg}\,\frac{x-4}{3} = \text{arc tg}\,\frac{x-4}{3} + c.$$

4. $$\int \frac{dx}{\sqrt{3x-x^2-2}}.$$

$$3x - x^2 - 2 = -x^2 + 3x - 2 = -(x^2 - 3x + 2)\ ;\ \frac{3}{2}\ ;\ \left(\frac{3}{2}\right)^2 = \frac{9}{4}.$$

$$-(x^2 - 3x + 2) = -(x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2) = -[(x - \frac{3}{2})^2 - \frac{9}{4} + \frac{8}{4}] =$$

$$= -\left\{\left(x-\frac{3}{2}\right)^2-\frac{1}{4}\right\} = -\left\{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2\right\} = \left(\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2$$

$$\int \frac{dx}{\sqrt{(1/2)^2-(x-3/2)^2}}.$$

$v = x - 3/2$
$dv = dx$
$a = 1/2$

Esta completo el diferencial. Se aplica: $\int \frac{dv}{\sqrt{a^2-v^2}} = \text{arc sen}\ \frac{v}{a} + c$.

$$= \text{arc sen}\left(\frac{x-3/2}{1/2}\right) = \text{arc sen}\left(\frac{\frac{2x-3}{\cancel{2}}}{1/\cancel{2}}\right) = \textbf{arc sen }(2x-3)+c\ .$$

5. $\int \frac{dv}{v^2-6v+5}.$

$v^2 - 6v + 5$; $\frac{6}{2} = 3$; $3^2 = 9$

$$v^2-6v+5 = v^2-6v+9-9+5 = (v-3)^2-4 = (v-3)^2-2^2 =$$

Sustituyendo este valor en la integral:

$$\int \frac{dv}{(v-3)^2-2^2}.$$

$v = v - 3$
$dv = dv$
$a = 2$

El diferencial esta completo, se emplea la fórmula: $\int \frac{dv}{v^2-a^2} = \frac{1}{2a}\ .\ \ln\frac{v-a}{v+a} + c$.

$$\int \frac{dv}{(v-3)^2-2^2} = \frac{1}{2.2}\ .\ \ln\left(\frac{v-3-2}{v-3+2}\right) = \frac{\mathbf{1}}{\mathbf{4}}\ .\ \mathbf{ln}\left(\frac{\mathbf{v-5}}{\mathbf{v-1}}\right) + \mathbf{c}\ .$$

6. $\int \frac{dx}{2x^2-2x+1}.$

$2x^2 - 2x + 1 = 2(x^2 - x + \frac{1}{2})$; $\frac{1}{2}$; $\frac{1}{2}^2 = \frac{1}{4}$

$2(x^2 - x + \frac{1}{4} - \frac{1}{4} + \frac{1}{2}) = 2\{ (x - \frac{1}{2})^2 - \frac{1}{4} + \frac{1}{2} \} = 2\{(x - \frac{1}{2})^2 - \frac{1}{4} + \frac{2}{4} \}$

$2\{(x - \frac{1}{2})^2 + \frac{1}{4} \} = 2\{(x - \frac{1}{2})^2 + \frac{1^2}{2^2}\}$

El factor (2) por estar en el denominador, sale fuera de la integral como 1/2 .

$$\int \frac{dx}{2\{(x - \frac{1}{2})^2 + \frac{1^2}{2^2}\}} = \frac{1}{2} . \int \frac{dx}{\{(x - \frac{1}{2})^2 + \frac{1^2}{2^2}\}} =$$

$v = x - 1/2$
$dv = dx$
$a = 1/2$

El diferencial esta completo. Se aplica: $\int \frac{dv}{v^2 + a^2} = \frac{1}{a} \text{ arc tg } \frac{v}{a} + c$.

$$\frac{1}{2} . \frac{1}{\frac{1}{2}} \int \frac{dx}{\{(x - \frac{1}{2})^2 + \frac{1^2}{2^2}\}} = \frac{1}{2} . 2 \text{ .arc tg} \left(\frac{x - \frac{1}{2}}{\frac{1}{2}} \right) =$$

$$\frac{2}{2} \text{ arc tg} \left(\frac{\frac{2x - 1}{\cancel{2}}}{\frac{1}{\cancel{2}}} \right) = \textbf{arc tg (2x - 1) + c .}$$

7. $\int \frac{dx}{\sqrt{15 + 2x - x^2}}$.

$15 + 2x - x^2 = - x^2 + 2x + 15 = - (x^2 - 2x - 15)$; $\frac{2}{2} = 1$; $1^2 = 1$

$(x^2 - 2x + 1 - 1 - 15) = - \{(x - 1)^2 - 16 \} = - [(x - 1)^2 - 4^2] =$

$[4^2 - (x - 1)^2]$. Se reemplaza este valor en la integral.

$$\int \frac{dx}{\sqrt{15 + 2x - x^2}} = \int \frac{dx}{\sqrt{\{4^2 - (x - 1)^2\}}} =$$

$$\left.\begin{array}{l} v = x - 1 \\ dv = dx \\ a = 4 \end{array}\right\} \quad \begin{array}{l} \text{El diferencial esta completo,se usa la fórmula:} \\ \displaystyle\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen } \frac{v}{a} + c\ . \end{array}$$

$$\textbf{arc sen } \frac{\mathbf{x - 1}}{\mathbf{4}} + \mathbf{c}\ .$$

8. $$\int \frac{\mathbf{dx}}{\mathbf{x^2 + 2x}}\ .$$

$x^2 + 2x$; $2/2 = 1$; $1^2 = 1$. Se suma y resta 1 a: $x^2 + 2x$.

$x^2 + 2x = x^2 + 2x + 1 - 1 = [(x + 1)^2 - 1] = [(x + 1)^2 - 1^2]$.

$$\int \frac{dx}{\{(x + 1)^2 - 1^2\}}\ .$$

$$\left.\begin{array}{l} v = x + 1 \\ dv = dx \\ a = 1 \end{array}\right\} \begin{array}{l} \text{El diferencial esta completo. Se usa la fórmula:} \\ \displaystyle\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln\left(\frac{v - a}{v + a}\right) + c\ . \end{array}$$

$$\int \frac{dx}{\{(x + 1)^2 - 1^2\}} = \frac{1}{2.1} \ln\left(\frac{x + 1 - 1}{x + 1 + 1}\right) = \frac{\mathbf{1}}{\mathbf{2}} \textbf{ ln } \left(\frac{\mathbf{x}}{\mathbf{x + 2}}\right) + \mathbf{c}\ .$$

9. $$\int \frac{\mathbf{dx}}{\mathbf{4x - x^2}}\ .$$

$4x - x^2 = - x^2 + 4x = - (x^2 - 4x)$

$\frac{4}{2} = 2$; $2^2 = 4$

$= -(x^2 - 4x + 4 - 4) = = -\{(x-2)^2 - 4\} = -\{(x-2)^2 - 2^2\} =$

$\{2^2 - (x-2)^2\}$

$$\int \frac{dx}{\{2^2 - (x-2)^2\}}.$$

$\left.\begin{array}{l} v = x - 2 \\ dv = dx \\ a = 2 \end{array}\right\}$ El diferencial esta completo,se usa la fórmula: $\int \frac{dv}{a^2 - v^2} = \frac{1}{2a}. \ln\left(\frac{a+v}{a-v}\right) + c$.

$$\frac{1}{2.2} \ln \frac{2 + x - 2}{2 - (x-2)} = \frac{1}{4} \ln\left(\frac{x}{2 - x + 2}\right) = \mathbf{\frac{1}{4} \ln \left(\frac{x}{4 - x}\right) + c}\ .$$

10. $\int \frac{\mathbf{dx}}{\sqrt{\mathbf{2x - x^2}}}$.

$2x - x^2 = -x^2 + 2x = -(x^2 - 2x)$; $\frac{2}{2} = 1$; $1^2 = 1$

$-(x^2 - 2x + 1 - 1) = \{-(x-1)^2 - 1\} = \{-(x-1)^2 - 1^2\} = 1^2 - (x-1)^2$

$$\int \frac{dx}{\sqrt{1^2 - (x-1)^2}}.$$

$\left.\begin{array}{l} v = x - 1 \\ dv = dx \\ a = 1 \end{array}\right\}$ Esta completo el diferencial, se usa la fórmula: $\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen } \frac{v}{a} + c$.

$\text{arc sen } \frac{x-1}{1} = \mathbf{arc\ sen\ (x - 1) + c}$.

11. $\int \frac{\mathbf{ds}}{\sqrt{\mathbf{2as + s^2}}}$.

$2as + s^2 = s^2 + 2as. \qquad \frac{2a}{2} = a \quad ; \quad a^2 = a^2$

$s^2 + 2as + a^2 - a^2 = \{(s + a)^2 - a^2\} = (s + a)^2 - a^2$

$$\int \frac{ds}{\sqrt{\{(s + a)^2 - a^2\}}}.$$

$\left.\begin{array}{l} v = s + a \\ dv = ds \\ a = a \end{array}\right\}$ El diferencial esta completo, se aplica: $\int \frac{dv}{\sqrt{v^2 - a^2}} = \ln [v + \sqrt{(v^2 - a^2)}] + c$.

$$\mathbf{\ln \{(s + a) + \sqrt{[(s + a)^2 - a^2]}\} + c.}$$

12. $\displaystyle\int \frac{\mathbf{dy}}{\mathbf{y^2 + 3y + 1}}.$

$y^2 + 3y + 1. \quad \frac{3}{2} ; \left(\frac{3}{2}\right)^2 = \frac{9}{4}.$

$y^2 + 3y + \frac{9}{4} - \frac{9}{4} + 1 = \{(y + \frac{3}{2})^2 - \frac{9}{4} + \frac{4}{4}\} = \{(y + \frac{3}{2})^2 - \frac{5}{4}\}.$

$\{(y + \frac{3}{2})^2 - \left(\frac{\sqrt{5}}{\sqrt{4}}\right)^2\} = \{(y + \frac{3}{2})^2 - \left(\frac{\sqrt{5}}{2}\right)^2\}$

$\displaystyle\int \frac{dy}{(y + 3/2)^2 - (\sqrt{5/2})^2}. \qquad \left.\begin{array}{l} v = y + 3/2 \\ dv = dy \\ a = \sqrt{5/2} \end{array}\right\}$ El diferencial esta completo, se aplica : $\displaystyle\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln \left(\frac{v - a}{v + a}\right) + c$

$$\frac{1}{\cancel{2}.\frac{\sqrt{5}}{\cancel{2}}} . \ln \left(\frac{y + \frac{3}{2} - \frac{\sqrt{5}}{2}}{y + \frac{3}{2} + \frac{\sqrt{5}}{2}}\right) = \frac{1}{\sqrt{5}} \ln \left(\frac{\frac{2y + 3 - \sqrt{5}}{\cancel{2}}}{\frac{2y + 3 + \sqrt{5}}{\cancel{2}}}\right) =$$

$$\frac{1}{\sqrt{5}} \ln \left(\frac{2y+3-\sqrt{5}}{2y+3+\sqrt{5}}\right) + c .$$

13. $\int \frac{dy}{1+x+x^2}$.

$1 + x + x^2 = x^2 + x + 1 .\ \frac{1}{2} ;\ \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

$= \{x^2 + x + \frac{1}{4} - \frac{1}{4} + 1\} = \{(x + ½)^2 - \frac{1}{4} + \frac{4}{4}\} =$

$\{(x + ½)^2 + ¾\} = \{(x + ½)^2 + (\sqrt{3}/4)^2\} = (x + ½)^2 + (\sqrt{3}/2)^2$.

$$\int \frac{dy}{(x+½)^2 + (\sqrt{3}/2)^2} =$$

$$\left.\begin{array}{l} v = x + 1/2 \\ dv = dx \\ a = \sqrt{3}/2 \end{array}\right\} \begin{array}{l} \text{El diferencial esta completo.} \\ \int \frac{dv}{v^2 + a^2} = \frac{1}{a} \text{arc tg} \frac{v}{a} + c . \end{array}$$

$$\int \frac{dy}{(x+½)^2 + (\sqrt{3}/2)^2} = \left(\frac{1}{\frac{\sqrt{3}}{2}}\right) \text{arc tg} \left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) =$$

$$\frac{2}{\sqrt{3}} \text{arc tg} \left(\frac{\frac{2x+1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}}\right) = \frac{2}{\sqrt{3}} \textbf{arc tg} \left(\frac{2x+1}{\sqrt{3}}\right) + c$$

14. $\int \frac{dx}{\sqrt{1+x+x^2}}$.

$$1 + x + x^2 = x^2 + x + 1 \ . \ \frac{1}{2} \ ; \ \left(\frac{1}{2}\right)^2 = \frac{1}{4} \ .$$

$$x^2 + x + \frac{1}{4} - \frac{1}{4} + 1 = \{(x + ½)^2 - \frac{1}{4} + \frac{4}{4}\} = \{(x + ½)^2 + ¾\} \ .$$

$$\left\{(x + ½)^2 + \left(\frac{\sqrt{3}}{\sqrt{4}}\right)^2\right\} = \left\{(x + ½)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right\} = \left\{(x + ½)^2 + (\sqrt{3}/2)^2\right\}$$

$$\int \frac{dx}{\sqrt{\{(x + ½)^2 + (\sqrt{3}/2)^2\}}} \ .$$

$v = x + 1/2$ | Esta completo el diferencial.
$dv = dx$ | Se aplica : $\int \frac{dv}{\sqrt{v^2+a^2}} = \ln\{v + \sqrt{v^2+a^2}\} + c.$
$a = \sqrt{3}/2$

$$\ln\{x + ½ + \sqrt{\{(x + ½)^2 + (\sqrt{3}/2)^2\}} =$$

$$\mathbf{\ln\{x + ½ + \sqrt{(1 + x + x^2)}\} + c \ .}$$

15. $$\int \frac{\mathbf{dx}}{\mathbf{4x^2 + 4x + 5}} \ .$$

$$4x^2 + 4x + 5 = 4(x^2 + x + \frac{5}{4}) \ . \quad \frac{1}{2} \ ; \ \frac{1^2}{2^2} = \frac{1}{4} .$$

$$4(x^2 + x + \frac{1}{4} - \frac{1}{4} + \frac{5}{4}) = 4(x^2 + x + \frac{1}{4} + \frac{4}{4}) =$$

$$4\{(x + ½)^2 + 1\} = 4\{(x + ½)^2 + 1^2\}.$$

El factor (4) sale como ¼ fuera de la integral

$$\frac{1}{4} . \int \frac{dx}{\{(x + \frac{1}{2})^2 + 1^2\}} \ .$$

$$\left.\begin{array}{l} v = x + 1/2 \\ dv = dx \\ a = 1 \end{array}\right\}$$ El diferencial esta completo:
Se aplica: $\int \frac{dv}{v^2 + a^2} = \frac{1}{a} \text{ arc tg } \frac{v}{a} + c$

$$\frac{1}{4} \cdot \frac{1}{1} \text{ arc tg } \frac{x + \frac{1}{2}}{1} = \frac{1}{4} \textbf{ arc tg } \frac{(2x + 1)}{2} + c \text{ .}$$

16. $\int \frac{dx}{3x^2 - 2x + 4}$.

$$3x^2 - 2x + 4 = 3(x^2 - 2/3x + 4/3). \quad \frac{\frac{2}{3}}{\frac{2}{1}} = \frac{2}{6} = \frac{1}{3} \quad ; \quad \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$3[x^2 - 2/3x + 1/9 - 1/9 + 4/3] = 3[(x - 1/3)^2 - 1/9 + 12/9] =$$

$$3[(x - 1/3)^2 + 11/9] = \{3(x - 1/3)^2 + (\sqrt{11}/\sqrt{9})^2 =$$

$$\{3(x - 1/3)^2 + (\sqrt{11}/3)^2\}$$

El factor (3) del denominador, sale como 1/3 fuera de la integral .

$$\int \frac{dx}{\{3(x - 1/3)^2 + (\sqrt{11}/3)^2\}} = \frac{1}{3} \int \frac{dx}{(x - 1/3)^2 + (\sqrt{11}/3)^2} \text{ .}$$

$$\left.\begin{array}{l} v = x - 1/3 \\ dv = dx \\ a = \sqrt{11}/3 \end{array}\right\}$$ El diferencial esta completo, se aplica:
$\int \frac{dv}{v^2 + a^2} = \frac{1}{a} \text{ arc tg } \frac{v}{a} + c$.

$$= \left\{\frac{1}{\cancel{3}} \cdot \frac{1}{\frac{\sqrt{11}}{\cancel{3}}}\right\} \text{ arc tg } \left\{\frac{x - \frac{1}{3}}{\frac{\sqrt{11}}{3}}\right\} = \left\{\frac{1}{\sqrt{11}}\right\} \text{arc tg } \left\{\frac{\frac{3x - 1}{\cancel{3}}}{\frac{\sqrt{11}}{\cancel{3}}}\right\} =$$

$$\left\{\frac{1}{\sqrt{11}}\right\}.\ \text{arc tg}\left\{\frac{3x-1}{\sqrt{11}}\right\} + c\ .$$

17. $\displaystyle\int \frac{dx}{\sqrt{2-3x-4x^2}}$.

$2 - 3x - 4x^2 = \{-4x^2 - 3x + 2\} = \{-4(x^2 + \tfrac{3}{4}x - 2/4)\}$,

$\left\{\dfrac{3/4}{2} = 3/8\ ;\ (3/8)^2 = 9/64\right\}$

$\{-4(x^2 + \tfrac{3}{4}x + 9/64 - 9/64 - 2/4)\} = \{-4[(x + 3/8)^2 - 9/64 - 32/64]\}$

$-4[(x + 3/8)^2 - 41/64]\} = \{-4[(x + 3/8)^2 - (\sqrt{41}/\sqrt{64})^2]\}$

$\{-4[(x + 3/8)^2 - (\sqrt{41}/8)^2]\} = \{4[(\sqrt{41}/8)^2 - (x + 3/8)^2]\} =$

Al factor (4) se le extrae la raiz cuadrada y sale fuera de la integral como ½

$$\int \frac{dx}{\sqrt{\{4[(\sqrt{41}/8)^2 - (x + 3/8)^2]\}}} = \int \frac{dx}{\sqrt{4}\ .\ \sqrt{[(\sqrt{41}/8)^2 - (x + 3/8)^2]}} =$$

$$\int \frac{dx}{2a\sqrt{[(\sqrt{41}/8)^2 - (x + 3/8)^2]}} = \frac{1}{2}\int \frac{dx}{\sqrt{[(\sqrt{41}/8)^2 - (x + 3/8)^2]}} =$$

$\left.\begin{array}{l} v = x + 3/8 \\ dv = dx \\ a = \sqrt{41/8} \end{array}\right\}$ El diferencial esta completo, se procede a integrar.
Se aplica : $\displaystyle\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen}\ \frac{v}{a} + c$.

$$\left(\frac{1}{2}\right)\text{arc sen}\left(\frac{x + 3/8}{\sqrt{41}/8}\right) = \frac{1}{2}\ \text{arc sen}\left(\frac{\frac{8x+3}{8}}{\frac{\sqrt{41}}{8}}\right) + c\ .$$

$$\frac{1}{2}\ \text{arc sen}\left(\frac{8x+3}{\sqrt{41}}\right) + c\ .$$

18. $\displaystyle\int \frac{dx}{x^2 + 2x + 10}$.

$x^2 + 2x + 10$, $2/2 = 1$; $1^2 = 1$

$x^2 + 2x + 1 - 1 + 10 = (x + 1)^2 - 1 + 10 = (x + 1)^2 + 9 =$

$(x + 1)^2 + 3^2$. Sustituyendo este valor en la integral.

$$\int \frac{dx}{(x+1)^2 + 3^2}\ .$$

$\left.\begin{array}{l} v = x + 1 \\ dv = dx \\ a = 3 \end{array}\right\}$ El diferencial esta completo. Se aplica: $\displaystyle\int \frac{dv}{v^2 + a^2} = \frac{1}{a}\ \text{arc tg}\ \frac{v}{a} + c$.

$$\int \frac{dx}{(x+1)^2 + 3^2} = \frac{1}{3}\ \text{arc tg}\left(\frac{x+1}{3}\right) + c\ .$$

19. $\displaystyle\int \frac{dx}{x^2 + 2x - 3}$.

$x^2 + 2x - 3$. $2/2 = 1$; $1^2 = 1$

$x^2 + 2x - 3 = x^2 + 2x + 1 - 1 - 3 = (x + 1)^2 - 4 = (x + 1)^2 - 2^2$

$$\int \frac{dx}{(x+1)^2 - 2^2}\ .$$

$\left.\begin{array}{l} v = x + 1 \\ dv = dx \\ a = 2 \end{array}\right\}$ El diferencial esta completo, se aplica: $\displaystyle\int \frac{dv}{v^2 - a^2} = \frac{1}{2a}\ \ln\left(\frac{v-a}{v+a}\right) + c$.

$$\frac{1}{2\,.\,2}\ln\left(\frac{x+1-2}{x+1+2}\right) = \frac{1}{4}\ln\left\{\frac{x-1}{x+3}\right\} + c\,.$$

20. $\displaystyle\int \frac{dy}{3-2y-y^2}.$

$3 - 2y - y^2 = -y^2 - 2y + 3 = -(y^2 + 2y - 3)\,.$

$2/2 = 1\,;\;\; 1^2 = 1$

$\{-(y^2 + 2y + 1 - 1 - 3)\} = \{-[(y+1)^2 - 1 - 3]\} = \{-[(y+1)^2 - 4]\}$

$\{-[(y+1)^2 - 2^2]\} = \{2^2 - (y+1)^2\}$. Sustituyendo en la integral.

$\displaystyle\int \frac{dy}{\{2^2 - (y+1)^2\}}.$

El diferencial esta completo, se aplica:

$$\left.\begin{array}{l} v = y+1 \\ dv = dy \\ a = 2 \end{array}\right\} \quad \int \frac{dv}{a^2 - v^2} = \frac{1}{2a}\ln\left(\frac{a+v}{a-v}\right) + c\,.$$

$$\frac{1}{2(2)}\ln\left(\frac{2+y+1}{2-(y+1)}\right) = \frac{1}{4}\ln\left(\frac{3+y}{2-y-1}\right) = \frac{1}{4}\ln\left(\frac{3+y}{1-y}\right) + c\,.$$

21. $\displaystyle\int \frac{3\,du}{\sqrt{5-4u-u^2}}\,.$

$5 - 4u - u^2 = -u^2 - 4u + 5 = -(u^2 + 4u - 5)\,.$

$\{4/2 = 2\,;\;\; 2^2 = 4\}$

$\{-(u^2 + 4u + 4 - 4 - 5)\} = \{-(u+2)^2 - 4 - 5\} = \{-(u+2)^2 - 9\}$

$\{-(u+2)^2 - 3^2\} = \{3^2 - (u+2)^2\}$ .Se reemplaza en la integral.

$$\int \frac{3\,du}{\sqrt{3^2-(u+2)^2}}\ .$$

$$\left.\begin{array}{l} v = u+2 \\ dv = du \\ a = 3 \end{array}\right\} \quad \text{El diferencial esta completo, se aplica: } \int \frac{dv}{\sqrt{a^2-v^2}} = \text{arc sen}\,\frac{v}{a} + c\ .$$

$$\int \frac{3\,du}{\sqrt{3^2-(u+2)^2}} = 3\int \frac{du}{\sqrt{3^2-(u+2)^2}} = \mathbf{3\ arc\ sen\ \frac{u+2}{3} + c\ .}$$

22. $$\int \frac{5\,dx}{\sqrt{x^2+2x+5}}\ .$$

$x^2 + 2x + 5$. $2/2 = 1$; $1^2 = 1$

$x^2 + 2x + 1 - 1 + 5 = (x+1)^2 - 1 + 5 = (x+1)^2 + 4$.

$(x+1)^2 + 2^2$. Sustituyendo este resultado en la integral.

$$\int \frac{5\,dx}{\sqrt{(x+1)^2+2^2}}\ .$$

$$\left.\begin{array}{l} v = x+1 \\ dv = dx \\ a = 2 \end{array}\right\} \quad \text{El diferencial esta completo, se aplica: } \int \frac{dv}{\sqrt{v^2+a^2}} = \ln\left(v + \sqrt{v^2+a^2}\right) + c\ .$$

$$\ln\{x+1+\sqrt{(x+1)^2+2^2}\} = \mathbf{\ln\{x+1+\sqrt{(x^2+2x+5)}\} + c\ .}$$

23. $$\int \frac{dx}{\sqrt{x^2+4x+3}}\ .$$

$x^2 + 4x + 3$. $\left\{4/2 = 2\ ;\ 2^2 = 4\right\}$

$x^2 + 4x + 4 - 4 + 3 = (x + 2)^2 - 4 + 3 = (x + 2)^2 - 1$.

$(x + 2)^2 - 1^2$. Este resultado se reemplaza en la integral.

$\int \frac{dx}{\sqrt{(x + 2)^2 - 1^2}}$. $\left.\begin{array}{l} v = x + 2 \\ dv = dx \\ a = 1 \end{array}\right\}$ El diferencial esta completo, se aplica: $\int \frac{dv}{\sqrt{v^2 - a^2}} = \ln [v + \sqrt{v^2 - a^2}\,] + c$.

$\mathbf{\ln \{ x + 2 + \sqrt{[(x + 2)^2 - 1^2]}\,\} + c}$ **.**

24. $\int \frac{\mathbf{dx}}{\mathbf{\sqrt{x^2 + 2x}}}$ **.**

$x^2 + 2x$. $\{2/2 = 1 \;;\; 1^2 = 1 \}$

$x^2 + 2x + 1 - 1 = (x + 1)^2 - 1 = (x + 1)^2 - 1^2$. Sustituyendo este valor en la integral

$\int \frac{dx}{\sqrt{(x + 1)^2 - 1^2}}$.

$\left.\begin{array}{l} v = x + 1 \\ dv = dx \\ a = 1 \end{array}\right\}$ El diferencial esta completo, se aplica: $\int \frac{dv}{\sqrt{v^2 - a^2}} = \ln [v + \sqrt{(v^2 - a^2)}\,] + c$.

$\mathbf{\ln \{x + 1 + \sqrt{[(x + 1)^2 - 1^2]}\,\} + c}$ **.**

25. $\int \frac{\mathbf{dt}}{\mathbf{\sqrt{3t - 2t^2}}}$ **.**

$3t - 2t^2 = - 2t^2 + 3t = -2(t^2 - 3/2.t)$.

$\frac{3/2}{2} = ¾ \quad ; \quad (¾)^2 = 9/16$

$\{-2(t^2 - 3/2.t + 9/16 - 9/16)\} = \{-2[(t - \tfrac{3}{4})^2 - 9/16)]\} =$

$2[9/16 - (t - \tfrac{3}{4})^2]\} = \{2[(3/4)^2 - (t - \tfrac{3}{4})^2]\}$.

$$\int \frac{dt}{\sqrt{\{2[(\tfrac{3}{4})^2 - (t - \tfrac{3}{4})^2]\}}} = \int \frac{dt}{\sqrt{(2)}.\sqrt{[(\tfrac{3}{4})^2 - (t - \tfrac{3}{4})^2]}} =$$

$$\frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{[(\tfrac{3}{4})^2 - (t - \tfrac{3}{4})^2]}}.$$

$\left.\begin{array}{l} v = t - \tfrac{3}{4} \\ dv = dt \\ a = \tfrac{3}{4} \end{array}\right\}$ El diferencial esta completo, se aplica: $\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen } \frac{v}{a} + c$.

$$\frac{1}{\sqrt{2}} \text{ arc sen } \left(\frac{t - \tfrac{3}{4}}{\tfrac{3}{4}}\right) = \frac{1}{\sqrt{2}} \text{arc sen} \left(\frac{\frac{(4t - 3)}{\cancel{4}}}{\frac{3}{\cancel{4}}}\right) = \frac{1}{\sqrt{2}} \text{ arc sen } \left(\frac{4t - 3}{3}\right) + c .$$

26. $\int \frac{\mathbf{dx}}{\mathbf{x^2 - 4x + 5}}$.

$x^2 - 4x + 5$. $4/2 = 2$; $2^2 = 4$

$x^2 - 4x + 5 = x^2 - 4x + 4 - 4 + 5 = (x - 2)^2 - 4 + 5 =$

$(x - 2)^2 + 1^2$ .Sustituyendo este valor en la integral. $\int \frac{dx}{(x - 2)^2 + 1}$.

$\left.\begin{array}{l} v = x - 2 \\ dv = dx \\ a = 1 \end{array}\right\}$ El diferencial esta completo, se aplica: $\int \frac{dv}{v^2 + a^2} = \frac{1}{a} \text{ arc tg } \frac{v}{a} + c$.

$$\frac{1}{1} \text{ arc tg } \left(\frac{x - 2}{1}\right) = \mathbf{arc\ tg\ (x - 2) + c} .$$

27. $\int \frac{dx}{2 + 2x - x^2}$.

$2 + 2x - x^2 = - x^2 + 2x + 2 = - (x^2 - 2x - 2)$.

$2/2 = 1$; $1^2 = 1$

$\{-(x^2 - 2x - 2)\} = \{-(x^2 - 2x + 1 - 1 - 2)\} = \{-[(x - 1)^2 - 1 - 2]\} =$

$\{-[(x - 1)^2 - 3]\} = \{-[(x - 1)^2 - (\sqrt{3})^2]\} = (\sqrt{3})^2 - (x - 1)^2$.

$\int \frac{dx}{(\sqrt{3})^2 - (x - 1)^2}$.

$\left.\begin{array}{l} v = x - 1 \\ dv = dx \\ a = \sqrt{3} \end{array}\right\}$ El diferencial esta completo, se aplica: $\int \frac{dv}{a^2 - v^2} = \frac{1}{2a} \ln\left(\frac{a + v}{a - v}\right) + c$.

$\frac{1}{2\sqrt{3}} \ln \left(\frac{\sqrt{3} + x - 1}{\sqrt{3} - (x - 1)}\right) = \frac{1}{2\sqrt{3}} \ln \left(\frac{\sqrt{3} + x - 1}{\sqrt{3} - x + 1}\right) + c$.

28. $\int \frac{dr}{r^2 - 2r - 3}$.

$r^2 - 2r - 3$. $\left\{\frac{2}{2} = 1 \quad ; \quad 1^2 = 1\right\}$

$r^2 - 2r - 3 = r^2 - 2r + 1 - 1 - 3 = (r - 1)^2 - 1 - 3 = (r - 1)^2 - 4 = (r - 1)^2 - 2^2$

Sustituyendo este valor en la integral. $\int \frac{dr}{(r - 1)^2 - 2^2}$.

$\left.\begin{array}{l} v = r - 1 \\ dv = dr \\ a = 2 \end{array}\right\}$ El diferencial esta completo, se aplica: $\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln\left(\frac{v - a}{v + a}\right) + c$.

$\frac{1}{2 . 2} . \ln\left(\frac{r - 1 - 2}{r - 1 + 2}\right) = \frac{1}{4} \ln\left(\frac{r - 3}{r + 1}\right) + c$.

29. $\displaystyle\int \frac{4\,dx}{\sqrt{x^2 - 4x + 13}}$.

$x^2 - 4x + 13$. $\{4/2 = 2\ ;\ 2^2 = 4\}$

$x^2 - 4x + 13 = x^2 - 4x + 4 - 4 + 13 = (x + 2)^2 - 4 + 13 =$

$(x + 2)^2 + 9 = (x + 2)^2 + 3^2$. Reemplazando en la integral.

$\displaystyle\int \frac{4\,dx}{\sqrt{(x + 2)^2 + 3^2}}$.

$v = x + 2$
$dv = dx$
$a = 3$

El diferencial esta completo, se aplica:

$\displaystyle\int \frac{dv}{\sqrt{v^2 + a^2}} = \ln\left[v + \sqrt{v^2 + a^2}\right] + c$.

$\mathbf{\ln\{x + 2 + \sqrt{[(x + 2)^2 + 3^2]}\} + c}$ **.**

30. $\displaystyle\int \frac{dz}{\sqrt{3 + 2z - z^2}}$.

$3 + 2z - z^2 = - z^2 + 2z + 3 = -(z^2 - 2z - 3)$. $\{2/2 = 1\ ;\ 1^2 = 1\}$

$\{-(z^2 - 2z - 3)\} = \{-(z^2 - 2z + 1 - 1 - 3)\} = \{-[(z - 1)^2 - 1 - 3]\} =$

$\{-[(z - 1)^2 - 4]\} = \{-[(z - 1)^2 - 2^2]\} = 2^2 - (z - 1)^2$

$\displaystyle\int \frac{dz}{\sqrt{2^2 - (z - 1)^2}}$.

$v = z - 1$
$dv = dz$
$a = 2$

El diferencial esta completo, se aplica:

$\displaystyle\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen}\,\frac{v}{a} + c$.

$\mathbf{\text{arc sen}\left(\frac{z - 1}{2}\right) + c}$ **.**

31. $\displaystyle\int \frac{dv}{\sqrt{v^2 - 8v + 15}}$.

$v^2 - 8v + 15$. $\left\{ 8/2 = 4 \ ; \ 4^2 = 16 \right\}$

$v^2 - 8v + 16 - 16 + 15 = (v - 4)^2 - 16 + 15 = (v - 4)^2 - 1 =$

$(v - 4)^2 - 1^2$. Reemplazando este valor en la integral.

$\displaystyle\int \frac{dv}{\sqrt{(v - 4)^2 - 1^2}}$.

$\left.\begin{array}{l} v = v - 4 \\ dv = dv \\ a = 1 \end{array}\right\}$ Esta completo el diferencial, se aplica: $\displaystyle\int \frac{dv}{\sqrt{v^2 - a^2}} = \ln (v + \sqrt{v^2 - a^2}\,) + c$.

$\mathbf{\ln \{v - 4 + \sqrt{[(v - 4)^2 - 1^2]}\} + c}$ **.**

32. $\displaystyle\int \frac{x\,dx}{x^4 - x^2 - 1}$.

$x^4 - x^2 - 1 = (x^2)^2 - x^2 - 1$. $\left\{ 1/2 \ ; \quad (1/2)^2 = \frac{1}{4} \right\}$

$(x^2)^2 - x^2 - 1 = (x^2)^2 - x^2 + \frac{1}{4} - \frac{1}{4} - 1 = (x^2 - \frac{1}{2})^2 - \frac{1}{4} - 1 =$

$(x^2 - \frac{1}{2})^2 - 5/4 = (x^2 - \frac{1}{2})^2 - (\sqrt{5}/\sqrt{4})^2 = (x^2 - \frac{1}{2})^2 - (\sqrt{5/2})^2 =$

$(x^2 - \frac{1}{2})^2 - (\sqrt{5}/2)^2$ .reemplazando este valor en la integral.

$\displaystyle\int \frac{x\,dx}{(x^2 - \frac{1}{2})^2 - (\sqrt{5}/2)^2}$.

$\left.\begin{array}{l} v = x^2 - \frac{1}{2} \\ dv = 2x\,dx \end{array}\right\}$ Falta (2) para completar
Se aplica:

$a = \frac{\sqrt{5}}{2}$ $\quad \int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln\left(\frac{v - a}{v + a}\right) + c .$

$$\left(\frac{1}{2}\right)\int \frac{(2)\,x\,dx}{(x^2 - ½)^2 - (\sqrt{5}/2)^2} .$$

$$\left(\frac{1}{2} \cdot \frac{1}{\frac{\cancel{2} \cdot \sqrt{5}}{\cancel{2}}}\right) . \ln\left(\frac{x^2 - \frac{1}{2} - \frac{\sqrt{5}}{2}}{x^2 - \frac{1}{2} + \frac{\sqrt{5}}{2}}\right) = \left(\frac{1}{2\sqrt{5}}\right) \ln \left(\frac{\frac{2x^2 - 1 - \sqrt{5}}{\cancel{2}}}{\frac{2x^2 - 1 + \sqrt{5}}{\cancel{2}}}\right) =$$

$$\left(\frac{1 \cdot \sqrt{5}}{2\sqrt{5} \cdot \sqrt{5}}\right) . \ln\left(\frac{2x^2 - 1 - \sqrt{5}}{2x^2 - 1 + \sqrt{5}}\right) = \left(\frac{\sqrt{5}}{10}\right) \ln \left(\frac{2x^2 - 1 - \sqrt{5}}{2x^2 - 1 + \sqrt{5}}\right) + c .$$

33. $\displaystyle \int \frac{dt}{\sqrt{1 - t - 2t^2}} .$

$$1 - t - 2t^2 = - 2t^2 - t + 1 = -2(t^2 + ½\, t - ½) .$$

$$\left\{ \frac{½}{2} = ¼ \;\; ; \; (¼)^2 = 1/16 \right\}$$

$$\{-2(t^2 + ½\, t - ½)\} = \{-2(t^2 + ½\, t + 1/16 - 1/16 - ½)\} =$$

$$\{-2[(t + ¼)^2 - 1/16 - ½]\} = \{-2[(t + ¼)^2 -1/16 - 8/16]\} =$$

$$\{-2[(t + ¼)^2 - 9/16]\} = \{-2[(t + ¼)^2 - (\sqrt{9}/\sqrt{16})^2]\}$$

$$\{2(-1)[(t + ¼)^2 - (¾)^2]\} = \{2[(¾)^2 - (t + ¼)^2]\} .$$

$$\int \frac{dt}{\sqrt{\{2[(¾)^2 - (t + ¼)^2]\}}} = \int \frac{dt}{\sqrt{2}\, \sqrt{[(¾)^2 - (t + ¼)^2]}} = \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{[(¾)^2 - (t + ¼)^2]}}$$

$$\left.\begin{array}{l} v = t + \tfrac{1}{4} \\ dv = dt \\ a = \tfrac{3}{4} \end{array}\right\} \text{El diferencial esta completo, se aplica: } \int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen } \frac{v}{a} + c .$$

$$\frac{1}{\sqrt{2}} \text{ arc sen } \left(\frac{t + \frac{1}{4}}{\frac{3}{4}}\right) = \left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}\right) \text{arc sen} \left(\frac{\frac{4t+1}{\cancel{4}}}{\frac{3}{\cancel{4}}}\right) = \frac{\sqrt{2}}{3} \textbf{ arc sen } \left(\frac{4t+1}{3}\right) + c .$$

34. $\displaystyle \int \frac{dx}{3x^2 + 4x + 1} .$

$3x^2 + 4x + 1 = 3(x^2 + 4/3x + 1/3). \left\{ \frac{4/3}{2} = 4/6 = 2/3 \;;\; (2/3)^2 = 4/9 \right\}$

$3(x^2 + 4/3x + 4/9 - 4/9 + 1/3) = 3\{(x + 2/3)^2 - 4/9 + 1/3) =$

$3\{(x + 2/3)^2 - 4/9 + 3/9) = = 3\{(x + 2/3)^2 - 1/9\} =$

$3\{(x + 2/3)^2 - (\sqrt{1}/\sqrt{9})^2\} = 3\{(x + 2/3)^2 - (1/3)^2\} .$

$$\int \frac{dx}{3x^2 + 4x + 1} = \int \frac{dx}{3\{(x + 2/3)^2 - (1/3)^2\}} = \frac{1}{3} \int \frac{dx}{(x + 2/3)^2 - (1/3)^2} =$$

$$\left.\begin{array}{l} v = x + 2/3 \\ dv = dx \\ a = 1/3 \end{array}\right\} \text{El diferencial esta completo, se aplica: } \int \frac{dv}{v^2 - a^2} = \frac{1}{2a} . \ln\left(\frac{v - a}{v + a}\right) + c .$$

$$\left(\frac{1}{3} \cdot \frac{1}{2 \cdot \frac{1}{3}}\right) . \ln \left(\frac{x + 2/3 - 1/3}{x + 2/3 + 1/3}\right) = \frac{1}{\frac{6}{3}} \ln \left(\frac{x + 1/3}{x + 3/3}\right) = \frac{1}{2} \ln \left(\frac{\frac{3x+1}{\cancel{3}}}{\frac{3x+3}{\cancel{3}}}\right) =$$

$$\frac{1}{2} \ln \left(\frac{3x + 1}{3x + 3}\right) = \textbf{ln} \left(\frac{\mathbf{3x + 1}}{\mathbf{3x + 3}}\right)^{\mathbf{1/2}} + \mathbf{c} .$$

35. $\displaystyle\int \frac{dw}{2w^2 + 2w + 1}$.

$2w^2 + 2w + 1 = 2(w^2 + w + \frac{1}{2})$. $\quad 1/2$; $(1/2)^2 = \frac{1}{4}$.

$\{2(w^2 + w + \frac{1}{4} - \frac{1}{4} + \frac{1}{2})\} = \{2(w + \frac{1}{2})^2 - \frac{1}{4} + \frac{1}{2}\} =$

$\{2(w + \frac{1}{2})^2 - \frac{1}{4} + 2/4\} = = \{2[(w + \frac{1}{2})^2 + \frac{1}{4}]\} =$
$\{2(w + \frac{1}{2})^2 + [\sqrt{(\frac{1}{4})^2}\,]\} =$

$\{2[(w + \frac{1}{2})^2 + (\frac{1}{2})^2]\}$ .Reemplazando en la integral.

$$\int \frac{dw}{\{2[(w + \frac{1}{2})^2 + (\frac{1}{2})^2]\}} = \frac{1}{2}\int \frac{dw}{(w + \frac{1}{2})^2 + (\frac{1}{2})^2} .$$

$\left.\begin{array}{l} v = w + \frac{1}{2} \\ dv = dw \\ a = \frac{1}{2} \end{array}\right\}$ El diferencial esta completo, se aplica: $\displaystyle\int \frac{dv}{v^2 + a^2} = \frac{1}{a}\,\text{arc tg}\left(\frac{v}{a}\right) + c$.

$$\left(\frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\right)\text{arc tg}\left(\frac{w + \frac{1}{2}}{\frac{1}{2}\quad 2}\right) = \frac{1}{\frac{1}{2}}\,\text{arc tg}\left(\frac{\frac{2w+1}{\cancel{2}}}{\frac{1}{\cancel{2}}}\right) =$$

arc tg (2w + 1) + c .

36. $\displaystyle\int \frac{x^2\,dx}{9x^6 - 3x^3 - 1}$.

$9x^6 - 3x^3 - 1$. Suponiendo que: $\left.\begin{array}{r} x^3 = m \\ x^6 = m^2 \end{array}\right\}$ □ $9m^2 - 3m - 1 =$

$9(m^2 - 3/9m - 1/9) = 9(m^2 - 1/3m - 1/9)$.

$\frac{1/3}{2}$; $\left(\frac{1}{6}\right)^2 = \frac{1}{36}$.

$\{9(m^2 - 1/3m + 1/36 - 1/36 - 1/9)\} = \{9[(m - 1/6)^2 - 1/36 - 1/9]\}$

$\{9[(m - 1/6)^2 - 1/36 - 4/36]\} = \{9[(m - 1/6)^2 - 5/36]\} =$

$\{9[(m - 1/6)^2 - (\sqrt{5}/\sqrt{36})^2]\} = \{9[(m - 1/6)^2 - (\sqrt{5}/6)^2]\}$.

Pero: $m = x^3$, sustituyendo :

$\{9[(m - 1/6)^2 - (\sqrt{5}/6)^2]\} = \{9[(x^3 - 1/6)^2 - (\sqrt{5}/6)^2]\}$.

$$\int \frac{x^2\,dx}{\{9[(x^3 - 1/6)^2 - (\sqrt{5}/6)^2]\}} = \frac{1}{9}\int \frac{x^2\,dx}{[(x^3 - 1/6)^2 - (\sqrt{5}/6)^2]\}} =$$

$$\left.\begin{array}{l} v = x^3 - 1/6 \\ dv = 3x^2\,dx \\ a = \sqrt{5}/6 \end{array}\right\}$$

Falta (3) para completar el diferencial.

Se aplica: $\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln \left(\frac{v - a}{v + a}\right) + c$.

$$\frac{1}{9}\cdot\frac{1}{3}\int \frac{(3)x^2\,dx}{(x^3 - 1/6)^2 - (\sqrt{5}/6)^2} =$$

$$\left(\frac{1}{27}\cdot\frac{1}{2.\frac{\sqrt{5}}{6}}\right) \ln \left(\frac{x^3 - 1/6 - \sqrt{5}/6}{x^3 - 1/6 + \sqrt{5}/6}\right) = \left(\frac{1}{54\,.\,\frac{\sqrt{5}}{6}}\right) \ln \left(\frac{\frac{6x^3 - 1 - \sqrt{5}}{\cancel{6}}}{\frac{6x^3 - 1 + \sqrt{5}}{\cancel{6}}}\right)$$

$$\left(\frac{1}{9\sqrt{5}}\right) \ln \left(\frac{6x^3 - 1 - \sqrt{5}}{6x^3 - 1 + \sqrt{5}}\right) = \left(\frac{1\,.\,\sqrt{5}}{9\sqrt{5}.\sqrt{5}}\right) \ln \left(\frac{6x^3 - 1 - \sqrt{5}}{6x^3 - 1 + \sqrt{5}}\right) =$$

$$\left(\frac{\sqrt{5}}{9\,.\,5}\right) \ln \left(\frac{6x^3 - 1 - \sqrt{5}}{6x^3 - 1 + \sqrt{5}}\right) = \mathbf{\frac{\sqrt{5}}{45} \ln \left(\frac{6x^3 - 1 - \sqrt{5}}{6x^3 - 1 + \sqrt{5}}\right) + c} .$$

<u>Verificación del Ejercicio # 36, mediante la Diferenciación:</u>

$$\frac{d}{dx}\left\{\frac{\sqrt{5}}{45}\cdot \ln\left(\frac{6x^3-1-\sqrt{5}}{6x^3-1+\sqrt{5}}\right)\right\}=\frac{\sqrt{5}}{45}\cdot\frac{d}{dx}\ln\left(\frac{6x^3-1-\sqrt{5}}{6x^3-1+\sqrt{5}}\right).$$

$$\frac{\sqrt{5}}{45}\left(\frac{1}{\frac{6x^3-1-\sqrt{5}}{6x^3-1+\sqrt{5}}}\right)\cdot\frac{d}{dx}\left(\frac{6x^3-1-\sqrt{5}}{6x^3-1+\sqrt{5}}\right)$$

$$\left(\frac{\sqrt{5}}{45}\cdot\frac{6x^3-1+\sqrt{5}}{6x^3-1-\sqrt{5}}\right)\left(\frac{(6x^3-1+\sqrt{5})(18x^2)-(6x^3-1-\sqrt{5})(18x^2)}{(6x^3-1+\sqrt{5})^2}\right)$$

$$\left(\frac{\sqrt{5}}{45}\cdot\frac{(6x^3-1+\sqrt{5})}{6x^3-1-\sqrt{5}}\right)\left(\frac{108x^5-18x^2+18.\sqrt{5}.x^2-(108x^5-18x^2-18.\sqrt{5}.x^2)}{(6x^3-1+\sqrt{5})^2}\right)$$

$$\left(\frac{\sqrt{5}}{45}\cdot\frac{\cancel{(6x^3-1+\sqrt{5})}}{6x^3-1-\sqrt{5}}\right)\left(\frac{\cancel{108x^5}-\cancel{18x^2}+18.\sqrt{5}.x^2-\cancel{108x^5}+\cancel{18x^2}+18.\sqrt{5}.x^2)}{(6x^3-1+\sqrt{5})\cancel{(6x^3-1+\sqrt{5})}}\right)$$

$$\left(\frac{\sqrt{5}}{45}\cdot\frac{36.\sqrt{5}.x^2}{(6x^3-1-\sqrt{5})(6x^3-1+\sqrt{5})}\right)=\left(\frac{36.5.x^2}{45(6x^3-1-\sqrt{5})(6x^3-1+\sqrt{5})}\right)$$

$$\left(\frac{\cancel{180}\ x^2}{\cancel{45}\ \{(6x^3-1)-(\sqrt{5})\}\{(6x^3-1)+(\sqrt{5})\}}\right)=\left(\frac{4x^2}{(6x^3-1)^2-(\sqrt{5})^2}\right)$$

$$\frac{4x^2}{36x^6-12x^3+1-5}=\frac{4x^2}{36x^6-12x^3-4}=\frac{\cancel{4}\ x^2}{\cancel{4}(9x^6-3x^3-1)}.$$

$$\frac{x^2}{(9x^6-3x^3-1)}.$$

Como es una diferenciación, para comprobar si la integral esta bien desarrollada, por comodidad no fuimos colocando el dx, en el sitio correcto que le compete, lo hacemos en la parte final; podemos asumir, como el dx esta dividiendo, pasa ahora a multiplicar.

$$\frac{d}{dx}\left(\frac{x^2}{9x^6-3x^3-1}\right)=d\left(\frac{x^2}{9x^6-3x^3-1}\right)dx.$$

L.q.d.d. (Lo que se queria demostrar).

37. $\displaystyle\int \frac{dt}{15 + 4t - t^2}$.

$15 + 4t - t^2 = - t^2 + 4t + 15 = -(t^2 - 4t - 15)$. $4/2 = 2$; $2^2 = 4$

$-(t^2 - 4t + 4 - 4 - 15) = -[(t - 2)^2 - 4 - 15] = -[(t - 2)^2 - 19] =$

$[19 - (t - 2)^2] = (\sqrt{19})^2 - (t - 2)^2$ .Sustituyendo este valor en la integral.

$$\int \frac{dt}{(\sqrt{19})^2 - (t - 2)^2} .$$

$\left.\begin{array}{l} v = t - 2 \\ dv = dt \\ a = \sqrt{19} \end{array}\right\}$ El diferencial esta completo, se aplica: $\displaystyle\int \frac{dv}{a^2 - v^2} = \frac{1}{2a} \ln\left(\frac{a + v}{a - v}\right) + c$.

$$\left(\frac{1}{2.\sqrt{19}}\right) \ln \left(\frac{\sqrt{19} + t - 2}{\sqrt{19} - (t - 2)}\right) = \left(\frac{1 . \sqrt{19}}{2.\sqrt{19}.\sqrt{19}}\right) \ln \left(\frac{\sqrt{19} + t - 2}{\sqrt{19} - t + 2}\right) =$$

$$\left(\frac{\sqrt{19}}{2.19}\right) \ln \left(\frac{\sqrt{19} + t - 2}{\sqrt{19} - t + 2}\right) = \left(\frac{\sqrt{19}}{38}\right) \ln \left(\frac{\sqrt{19} + t - 2}{\sqrt{19} - t + 2}\right) + c .$$

38. $\displaystyle\int \frac{dx}{\sqrt{9x^2 + 12x + 8}}$.

$9x^2 + 12x + 8 = 9(x^2 + 12/9x + 8/9)$.

$\dfrac{12/9}{2} = 12/18 = 2/3$; $(2/3)^2 = 4/9$

$9(x^2 + 12/9x + 4/9 - 4/9 + 8/9) = 9[(x + 2/3)^2 - 4/9 + 8/9] =$

$9[(x + 2/3)^2 + 4/9] = 9[(x + 2/3)^2 + (\sqrt{4}/\sqrt{9})^2] = 9[(x + 2/3)^2 + (2/3)^2]$

Reemplazando este valor en la integral.

$$\int \frac{dx}{\sqrt{9(x + 2/3)^2 + (2/3)^2}} = \int \frac{dx}{\sqrt{9}.\sqrt{(x + 2/3)^2 + (2/3)^2}} =$$

$$\frac{1}{3}\int \frac{dx}{\sqrt{(x + 2/3)^2 + (2/3)^2}}.$$

$$\left.\begin{matrix} v = x + 2/3 \\ dv = dx \\ a = 2/3 \end{matrix}\right\} \text{El diferencial esta completo, se aplica: } \int \frac{dv}{\sqrt{v^2 + a^2}} = \ln\left(v + \sqrt{v^2 + a^2}\right) + c .$$

$$\mathbf{\frac{1}{3} \ln \left\{ x + 2/3 + \sqrt{[(x + 2/3)^2 + (2/3)^2]}\right\} + c .}$$

39. $$\int \frac{\mathbf{dx}}{\sqrt{\mathbf{4x^2 - 12x + 7}}}.$$

$4x^2 - 12x + 7 = 4(x^2 - 3x + 7/4)$. $3/2$; $(3/2)^2 = 9/4$.

$\{4(x^2 - 3x + 9/4 - 9/4 + 7/4)\} = \{4[(x - 3/2)^2 - 9/4 + 7/4]\} =$

$\{4[(x - 3/2)^2 - 2/4]\} = \{4[(x - 3/2)^2 - (\sqrt{2}/\sqrt{4})]\}^2 =$

$\{4[(x - 3/2)^2 - (\sqrt{2}/2)^2]\}$. Reemplazando en la integral.

$$\int \frac{dx}{\sqrt{\{4[(x - 3/2)^2 - (\sqrt{2}/2)^2]\}}} = \int \frac{dx}{\sqrt{4}.\ \sqrt{[(x - 3/2)^2 - (\sqrt{2}/2)^2]}} =$$

$$\frac{1}{2}\int \frac{dx}{\sqrt{(x - 3/2)^2 - (\sqrt{2}/2)^2}} =$$

$$\left.\begin{matrix} v = x - 3/2 \\ dv = dx \\ a = \sqrt{2}/2 \end{matrix}\right\} \text{El diferencial esta completo, se aplica: } \int \frac{dv}{\sqrt{v^2 - a^2}} = \ln\left(v + \sqrt{v^2 - a^2}\right) + c .$$

$$\frac{1}{2} \ln \left\{ x - 3/2 + \sqrt{(x - 3/2)^2 - (\sqrt{2}/2)^2} \right\} + c .$$

∄ pag 101

Problemas. Paginas 253 y 254

Verificar las siguientes Integraciones:

1. $\displaystyle \int \frac{(1+2x)\,dx}{1+x^2} = \textbf{arc tg } x + \ln(1+x^2) + c\ .$

Primero tomamos como referencia un artificio aritmético cualquiera:

$$\frac{7+14}{3+4} = \frac{7}{3+4} + \frac{14}{3+4} \quad ; \quad \frac{1+2x}{1+x^2} = \frac{1}{1+x^2} + \frac{2x}{1+x^2}\ .$$

Aplicando este artificio en la integral:

$$\int\left(\frac{1}{1+x^2} + \frac{2x}{1+x^2}\right)dx = \int\frac{dx}{1+x^2} + \int\frac{2x\,dx}{1+x^2} =$$

$\left.\begin{array}{l} v = x \\ dv = dx \\ a = 1 \end{array}\right\}$ La 1ra integral, esta completa. Se aplica: $\displaystyle\int\frac{dv}{a^2+v^2} = \frac{1}{a}\text{ arc tg }\frac{v}{a} + c\ .$

$\left.\begin{array}{l} v = 1+x^2 \\ dv = 2x\,dx \end{array}\right\}$ La 2da integral, tambien esta completa. Se aplica: $\displaystyle\int\frac{dv}{v} = \ln v + c.$

$$\frac{1}{1}\text{ arc tg }\frac{x}{1} + \ln(1+x^2) = \textbf{arc tg } x + \ln(1+x^2) + c\ .$$

2. $\displaystyle \int \frac{(2x+1)\,dx}{\sqrt{x^2-1}}\ .$

$$\int\frac{2x}{\sqrt{x^2-1}} + \int\frac{dx}{\sqrt{x^2-1}}\ .$$

$$\int\frac{2x\,dx}{(x^2-1)^{1/2}} + \int\frac{dx}{\sqrt{x^2-1^2}} = \int(x^2-1)^{-1/2}.\,2x\,dx + \int\frac{dx}{\sqrt{x^2-1^2}} =$$

$\left.\begin{array}{l} v = (x^2-1) \\ dv = 2x\,dx \\ n = -1/2 \end{array}\right\}$ <u>1ra integral</u>.Esta completo el diferencial. Se aplica: $\displaystyle\int v^n\,dv = \frac{v^{n+1}}{n+1} + c\ .$

$v = x$
$dv = dx$
$a = 1$
2^da^ integral. Se aplica: $\int \frac{dx}{\sqrt{v^2 - a^2}} = \ln\left(v + \sqrt{v^2 - a^2}\right) + c$

$$\frac{(x^2 - 1)^{-1/2+1}}{-1/2 + 1} + \ln\{x + \sqrt{x^2 - 1^2}\} = \frac{(x^2 - 1)^{1/2}}{1/2} + \ln\{x + \sqrt{x^2 - 1^2}\} =$$

$$2(x^2 - 1)^{1/2} + \ln\{x + \sqrt{x^2 - 1^2}\} = \mathbf{2\sqrt{x^2 - 1^2} + \ln\{x + \sqrt{x^2 - 1^2}\} + c}.$$

3. $\displaystyle \int \frac{(x - 1)}{\sqrt{1 - x^2}}\,dx$ **.**

$$\int \frac{x\,dx}{\sqrt{1 - x^2}} - \int \frac{dx}{\sqrt{1 - x^2}} = \int (1 - x^2)^{-1/2}.\,x\,dx - \int \frac{dx}{\sqrt{1 - x^2}}.$$

$v = 1 - x^2$
$dv = -2x$
$n = -1/2$
1^ra^ integral. Falta (-2) para completar el diferencial. Se aplica: $\int v^n\,dv = \frac{v^{n+1}}{n+1} + c$.

$v = x$
$dv = dx$
$a = 1$
2^da^ integral. Esta completo el diferencial. Se aplica: $\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen}\,\frac{v}{a} + c . + c$.

$$\left(\frac{-1}{2}\right)\int (1 - x^2)^{-1/2}.(-2)\,x\,dx - \int \frac{dx}{\sqrt{1^2 - x^2}}.$$

$$\frac{-1}{2} \cdot \frac{(1 - x^2)^{-1/2+1}}{-1/2+1} - \text{arc sen}\,\frac{x}{1} = -\frac{(1 - x^2)^{1/2}}{2(1/2)} - \text{arc sen}\,x =$$

$$-(1 - x^2)^{1/2} - \text{arc sen}\,x = \mathbf{-\sqrt{(1 - x^2)^{1/2}} - \text{arc sen}\,x + c}.$$

4. $\displaystyle \int \frac{(3x - 1)}{(x^2 + 9)}\,dx$ **.**

$$\int \frac{3x\,dx}{(x^2+9)} - \int \frac{dx}{(x^2+9)} = \int \frac{3x\,dx}{(x^2+3^2)} - \int \frac{dx}{(x^2+3^2)}.$$

$\left.\begin{array}{l} v = x^2 + 9 \\ dv = 2x\,dx \end{array}\right\}$ 1ra integral. $\qquad \left.\begin{array}{l} v = x \\ dv = dx \end{array}\right\}$ 2da integral.

1ra integral. Falta (2) para completar el diferencial, se aplica: $\int dv/v = \ln v + c$.Pero antes se coloca al # 3 fuera de la integral.

2da integral. Esta completo el diferencial, se aplica:

$$\int \frac{dv}{v^2+a^2} = \frac{1}{a}\text{arc tg}\,\frac{v}{a} + c\ .$$

$$3\,.\frac{1}{2}.\int \frac{(2)x\,dx}{(x^2+3^2)} - \int \frac{dx}{(x^2+3^2)} = \mathbf{\frac{3}{2}\ln(x^2+3^2) - \frac{1}{3}\,\text{arc tg}\,\frac{x}{3} + c}\ .$$

5. $\displaystyle \int \frac{(3s-2)\,ds}{\sqrt{9-s^2}}\ .$

$$\int \frac{3s}{\sqrt{9-s^2}} - 2\int \frac{ds}{\sqrt{9-s^2}} = \int \frac{3s}{(9-s^2)^{1/2}} - 2\int \frac{ds}{\sqrt{3^2-s^2}} =$$

$$3\int (9-s^2)^{-1/2}.sds\ - 2\int \frac{ds}{\sqrt{3^2-s^2}}.$$

$\left.\begin{array}{l} v = 9 - s^2 \\ dv = -2s\,ds \\ n = -1/2 \end{array}\right\}$ 1ra integral,falta (-2). Se aplica: $\int v^n\,dv = \frac{v^{n+1}}{n+1} + c$. $\qquad \left.\begin{array}{l} v = s \\ dv = ds \end{array}\right\}$ 2da integral esta completo el diferencial.

$$\int \frac{dv}{a^2-v^2} = = 3(-1/2)\int (9-s^2)^{-1/2}.(-2)\,sds\ - 2\int \frac{ds}{\sqrt{3^2-s^2}}.$$

$$\frac{-3}{}.\frac{(9-s^2)^{-1/2+1}}{} - 2\ \text{arc sen}\ \frac{s}{} = \frac{-3}{}.\ \frac{(9-s^2)^{1/2}}{} - 2\text{arc sen}\ \frac{s}{} =$$

2 -1/2+1 3 2 1/2 3

$$= \frac{-3.\cancel{2}}{\cancel{2}}.(9-s^2)^{1/2} - 2\text{arc sen}\,\frac{s}{3} = -3(9-s^2)^{1/2} - 2\text{arc sen}\,\frac{s}{3} =$$

$$= \mathbf{-3\sqrt{(9-s^2)} - 2\text{arc sen}\,\frac{s}{3} + c\ .}$$

6. $\displaystyle\int \frac{(x+3)}{\sqrt{x^2+4}}\,dx\ .$

$$\int \frac{x\,dx}{\sqrt{x^2+4}} + 3\int \frac{dx}{\sqrt{x^2+4}} = \int \frac{x\,dx}{(x^2+4)^{1/2}} + 3\int \frac{dx}{\sqrt{x^2+2^2}} =$$

$$\int (x^2+4)^{-1/2}.\,x\,dx + 3\int \frac{dx}{\sqrt{x^2+2^2}} =$$

$$\left.\begin{array}{l} v = x^2+4 \\ dv = 2x\,dx \\ n = -1/2 \end{array}\right\} \begin{array}{l} \underline{1^{ra}\text{ integral}}.\ \text{Falta (2) para completar el diferencial.} \\ \text{Se aplica: } \int v^n dv = \dfrac{v^{n+1}}{n+1} + c. \end{array}$$

$$\left.\begin{array}{l} v = x \\ dv = dx \\ a = 2 \end{array}\right\} \begin{array}{l} \underline{2^{da}}\text{ integral. Completo el diferencial. Se aplica:} \\ \int \dfrac{dv}{\sqrt{v^2+a^2}} = \ln[v + \sqrt{v^2+a^2}] + c\ . \end{array}$$

$$(1/2)\int (x^2+4)^{-1/2}.(2)\,x\,dx + 3\int \frac{dx}{\sqrt{x^2+2^2}} =$$

$$\frac{1}{2}.\frac{(x^2+4)^{-1/2+1}}{-1/2+1} + 3\,.\ln\{x + \sqrt{x^2+2^2}\} =$$

$$\frac{1}{2}.\frac{(x^2+4)^{1/2}}{1/2} + 3\ln\{x + \sqrt{x^2+4}\} =$$

$$(x^2+4)^{1/2} + 3\ln\{x + \sqrt{x^2+4}\} = \mathbf{(x^2+4)^{1/2} + 3\ln\{x + \sqrt{x^2+4}\} + c\ .}$$

7. $\displaystyle \int \frac{(2x-5)\,dx}{3x^2-2}$.

$$\int \frac{2x\ dx}{3x^2-2} - \int \frac{5dx}{3x^2-2} = \int \frac{2x\ dx}{3x^2-2} - 5\int \frac{dx}{3\left(x^2 - \frac{2}{3}\right)} =$$

$$\int \frac{2x\ dx}{3x^2-2} - 5\,.\,\frac{1}{3}\,.\int \frac{dx}{(x)^2 - \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2} =$$

$v = 3x^2 - 2$, $dv = 6x\ dx$ — <u>1^{ra} integral</u>. Falta (3) para completar el diferencial. Se aplica: $\int dv/v = \ln v + c$.

$v = x$, $dv = dx$, $a = \frac{\sqrt{2}}{\sqrt{3}}$ — <u>2^{da} integral</u>. Esta completo el diferencial. Se aplica: $\displaystyle \int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln \left\{ \frac{v-a}{v+a} \right\} + c$.

$$= \left(\frac{1}{3}\right)\int \frac{2(3)\,x\,dx}{3x^2-2} - \frac{5}{3}\int \frac{dx}{(x)^2 - \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2} =$$

$$\frac{1}{3}\,.\,\ln(3x^2-2) - \frac{5}{3}\,.\left\{\frac{1}{2.\ \frac{\sqrt{2}}{\sqrt{3}}}\right\}\ln\left\{\frac{x - \frac{\sqrt{2}}{\sqrt{3}}}{x + \frac{\sqrt{2}}{\sqrt{3}}}\right\} =$$

$$\frac{1}{3}\,.\,\ln(3x^2-2) - \left\{\frac{5\sqrt{3}}{6.\ \sqrt{2}}\right\}\ln\left\{\frac{x - \frac{\sqrt{2}\,.\,\sqrt{3}}{\sqrt{3}\,.\,\sqrt{3}}}{x + \frac{\sqrt{2}\,.\,\sqrt{3}}{\sqrt{3}\,.\,\sqrt{3}}}\right\} =$$

$$\frac{1}{3} \cdot \ln(3x^2 - 2) - \left\{\frac{5 . \sqrt{3} . \sqrt{2}}{6 . \sqrt{2} . \sqrt{2}}\right\} . \ln \left\{\frac{x - \frac{\sqrt{6}}{3}}{x + \frac{\sqrt{6}}{3}}\right\} =$$

$$\frac{1}{3} \cdot \ln(3x^2 - 2) - \left\{\frac{5 . \sqrt{6}}{6 . 2}\right\} . \ln \left\{\frac{\frac{3x - \sqrt{6}}{\cancel{3}}}{\frac{3x + \sqrt{6}}{\cancel{3}}}\right\} =$$

$$\frac{1}{3} \cdot \ln(3x^2 - 2) - \left\{\frac{5\sqrt{6}}{12}\right\} . \ln \left\{\frac{3x - \sqrt{6}}{3x + \sqrt{6}}\right\} + c .$$

8. $\int \frac{(5t - 1)\, dt}{\sqrt{3t^2 - 9}}$ **.**

$$= \int \frac{5\, t\, dt}{\sqrt{3t^2 - 9}} - \int \frac{dt}{\sqrt{[(\sqrt{3}.t)^2 - 3^2]}} = 5 \int \frac{t\, dt}{(3t^2 - 9)^{1/2}} - \int \frac{dt}{\sqrt{[(\sqrt{3}.t)^2 - 3^2]}}$$

$$5 \int (3t^2 - 9)^{-1/2} . t\, dt - \int \frac{dt}{\sqrt{[(\sqrt{3}.t)^2 - 3^2]}} =$$

$v = 3t^2 - 9$ Falta (6) para completar el diferencial.(1ra integral).

$dv = 6t\, dt$ Se aplica: $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$.

$n = -1/2$

$v = \sqrt{3}. t$ Falta ($\sqrt{3}$) para completar el diferencial.(2da integral).

$dv = \sqrt{3}$ Se aplica: $\int \frac{dv}{\sqrt{v^2 - a^2}} = \ln(v + \sqrt{v^2 - a^2}) + c$.

$a = 3$

$$5 . \frac{1}{6} . \int (3t^2 - 9)^{-1/2} .(6)\, t\, dt - \frac{1}{\sqrt{3}} \int \frac{\sqrt{3}\, dt}{\sqrt{[(\sqrt{3}.t)^2 - 3^2]}} =$$

$$= \frac{5}{6} . \frac{(3t^2 - 9)^{-1/2+1}}{-1/2 + 1} . - \frac{1}{\sqrt{3}} . \ln\{\sqrt{3}.t + [(\sqrt{3}.t)^2 - 3^2]\} =$$

Pero $:[\sqrt{(\sqrt{3.t})^2 - 3^2}] = \sqrt{3t^2 - 9}$, además ordenando $\sqrt{3}.t = t\sqrt{3}$

$$\square = \frac{5}{6}\left[\frac{(3t^2 - 9)^{1/2}}{1/2}\right] - \frac{1}{\sqrt{3}} . \ln\{t\sqrt{3} + \sqrt{3t^2 - 9}\}.$$

9. $\int \frac{(x + 3)\,dx}{6x - x^2}.$

Haciendo artificios con el númerador de la integral:
$x + 3 - 3 + 3 = \{x - 3 + 6\} = \{- 3 + x + 6\} = \{-(3 - x) + 6\}.$
Reemplazando en la integral.

$$\int \frac{\{-(3 - x) + 6\}\;dx}{6x - x^2} = -\int \frac{(3 - x)}{6x - x^2}dx + 6\int \frac{dx}{6x - x^2} =$$

Multiplicamos y dividimos para (2) al númerador de la 1^{ra} integral .

$$-\frac{1}{2}\int \frac{2(3 - x)}{6x - x^2}dx + 6\int \frac{dx}{6x - x^2} =$$

Descomponemos el denominador de la 2^{da} integral:

$6x - x^2 = - (x^2 - 6x)$. $6/2 = 3$; $3^2 = 9$

$- (x^2 - 6x + 9 - 9) = - \{(x - 3)^2 - 9 = -\{(x - 3)^2 - 3^2\}$. Este valor se sustituye en la 2^{da} integral.

$$-\frac{1}{2}\int \frac{2(3 - x)}{6x - x^2}dx + 6\int \frac{dx}{-\{(x - 3)^2 - 3^2\}} =$$

$$-\frac{1}{2}\int \frac{2(3 - x)}{6x - x^2}dx + 6\int \frac{dx}{(-)\{(x - 3)^2 - 3^2\}} =$$

Sacando el signo negativo (-) fuera de la integral como producto:

$$-\frac{1}{2}\int \frac{2(3 - x)}{6x - x^2}dx + \frac{6}{(-)}\int \frac{dx}{\{(x - 3)^2 - 3^2\}} =$$

$$-\frac{1}{2}\int \frac{2(3-x)}{6x-x^2}dx - 6\int \frac{dx}{\{(x-3)^2-3^2\}} = \_\_\_\_$$

$v = 6x - x^2$
$dv = 6 - 2x$
1ra Integral. El diferencial esta completo, al hacer operaciones: $2(3 - x) = 6 - 2x$, nos da el verdadero diferencial . Se aplica: $\int dv/v = \ln v + c$.

$v = x - 3$
$dv = dx$
$a = 3$
2da Integral. El diferencial esta completo. Se aplica: $\int \frac{dv}{v^2-a^2} = \frac{1}{2a}\ln\left(\frac{v-a}{v+a}\right) + c$.

$$-\frac{1}{2}\ln\{6x-x^2\} - 6 . \frac{1}{2 . 3} . \ln\left(\frac{x-3-3}{x-3+3}\right) + c . \_\_\_\_\ \_\_\_\_$$

$$-\frac{1}{2}\ln\{6x-x^2\} - \cancel{6} . \frac{1}{\cancel{6}} . \ln\left(\frac{x-6}{x}\right) + c . \_\_\_\_\ \_\_\_\_$$

$$-\frac{1}{2}\ln\{6x-x^2\} - \ln\left(\frac{x-6}{x}\right) + c . \_\_\_\_\ \_\_\_\_$$

10. $\int \frac{(2x+5)\,dx}{x^2+2x+5}$.

Suponiendo que: $v = x^2 + 2x + 5$; $dv = 2x + 2$.(verdadero diferencial)

Haciendo artificios: $(2x + 5)$ lo descomponemos en :

$(2x + 2 + 3)dx = [(2x + 2) + 3]dx$.

$$\int \frac{\{(2x+2)+3\}dx}{x^2+2x+5} = \int \frac{(2x+2)\ dx}{x^2+2x+5} + \int \frac{3\ dx}{x^2+2x+5} =$$

Descomponiendo: $x^2 + 2x + 5$. $\{2/2 = 1\ ;\ 1^2 = 1\}$

$\Box\ x^2 + 2x + 1\ \ - 1\ \ + 5 = x^2 + 2x + 1 + 4 = (x + 1)^2 + 2^2$.

Reemplazando este resultado en la $2^{\underline{da}}$ integral .

$$\int \frac{(2x + 2)\ dx}{x^2 + 2x + 5} + \int \frac{3\ dx}{x^2 + 2x + 5} =$$

$\left.\begin{array}{l} v = x^2 + 2x + 5 \\ dv = (2x + 2)\ dx \end{array}\right\}$ La $1^{\underline{ra}}$ integral, tiene el diferencial completo: Se aplica: $\int \frac{dv}{v} = \ln v + c$.

$\left\{\begin{array}{l}\text{La } 2^{\underline{da}} \text{ integral, tambien tiene el diferencial completo:} \\ \text{Se aplica: } \int \frac{dv}{v^2 + a^2} = \frac{1}{a}\ \text{arc tg}\ \frac{v}{a} + c\ .\end{array}\right.$

$$\int \frac{(2x + 2)\ dx}{x^2 + 2x + 5} + \int \frac{3\ dx}{x^2 + 2x + 5} = \int \frac{(2x + 2)\ dx}{x^2 + 2x + 5} + \int \frac{3\ dx}{(x + 1)^2 + 2^2}$$

$$\ln (x^2 + 2x + 5) + 3\ .\frac{1}{2}.\text{arc tg}\ \frac{(x + 1)}{2} =$$

$$\mathbf{\ln (x^2 + 2x + 5) + \frac{3}{2}\text{arc tg}\ \frac{(x + 1)}{2} + c\ .}$$

11. $\int \frac{(1 - x)\ dx}{4x^2 - 4x - 3}$.

$$\int \frac{-(-1 + x)\ dx}{4x^2 - 4x - 3} = -\int \frac{(x - 1)\ dx}{4x^2 - 4x - 3} = -\frac{1}{8}\int \frac{8(x - 1)\ dx}{4x^2 - 4x - 3}.$$

$$-\frac{1}{8}\int \frac{8x - 8\ dx}{4x^2 - 4x - 3} = -\frac{1}{8}\int \frac{(8x - 4 - 4)dx}{4x^2 - 4x - 3} = -\frac{1}{8}\int \frac{(8x - 4) - 4\ \ dx}{4x^2 - 4x - 3}$$

$$-\frac{1}{8}\left\{\int \frac{(8x - 4)\ dx}{4x^2 - 4x - 3} - \int \frac{\cancel{4}\ dx}{\cancel{4}\ (x^2 - x - 3/4)}\right\} =$$

$$-\frac{1}{8}\left\{\int \frac{(8x-4)\,dx}{4x^2-4x-3} - \int \frac{dx}{(x^2-x-3/4)}\right\}.$$

Descomponiendo: $x^2 - x - 3/4$, para hacerlo cuadrado perfecto.

$(x^2 - x - 3/4)\,.\ \frac{1}{2}$; $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

$(x^2 - x + 1/4 - 1/4 - 3/4) = (x^2 - x + 1/4 - 4/4) =$

$(x - 1/2) - 1 = (x - 1/2)^2 - 1^2$. Se reemplaza en la 2da integral.

$$-\frac{1}{8}\left\{\int \frac{(8x-4)\,dx}{4x^2-4x-3} - \int \frac{dx}{(x-1/2)^2-1^2}\right\}$$

La 1ra integral ,esta completa. $v = 4x^2 - 4x - 3$; $dv = 8x - 4$;
Se aplica : $\int \frac{dv}{v} = \ln v + c$.

La 2da integral , tambien esta completa
$v = x - 1/2$; $dv = dx$; $a = 1$.
Se aplica : $\int \frac{dv}{v^2-a^2} = \frac{1}{2a} \ln \frac{v-a}{v+a} + c$.

Integrando:

$$-\frac{1}{8}\left\{\ln(4x^2-4x-3) - \frac{1}{2.1}\ln\left[\frac{x-\frac{1}{2}-\frac{2}{2}}{x-\frac{1}{2}+\frac{2}{2}}\right]\right\} =$$

$$-\frac{1}{8}\left\{\ln(4x^2-4x-3) - \frac{1}{2}\ln\left[\frac{\frac{2x-1-2}{\cancel{2}}}{\frac{2x-1+2}{}}\right]\right\} =$$

$$-\frac{1}{8}\ln(4x^2 - 4x - 3) + \left(\frac{1}{8.2}\right)\ln\left(\frac{2x - 1 - 2}{2x - 1 + 2}\right)$$

$$-\frac{1}{8}\ln(4x^2 - 4x - 3) + \frac{1}{16}\ln\left(\frac{2x - 3}{2x + 1}\right) + c\,.$$

12. $\displaystyle\int \frac{(3x - 2)\,dx}{1 - 6x - 9x^2}$.

$$\int \frac{(3x - 2)\,dx}{-(9x^2 + 6x - 1)} = -\int \frac{(3x - 2)\,dx}{9x^2 + 6x - 1} =$$

Suponiendo que:
$v = 9x^2 + 6x - 1$; $dv = 18x + 6$;(verdadero diferencial); a :$(3x - 2)$ lo multiplicamos por (6) ; $6(3x - 2)dx = (18x - 12)dx$ y al mismo tiempo se le opone 1/6 a la integral.

Descomponiendo :
$9x^2 + 6x - 1 = 9(x^2 + 6/9x - 1/9) = 9(x^2 + 2/3x - 1/9)$.Se le extrae la mitad al coeficiente del 2$^{\underline{do}}$ término y al al resultado se lo eleva al cuadrado.

Luego, se suma y resta el resultado 1/9 : a $(x + 2/3x - 1/9)$.

$$\frac{2/3}{2} = \frac{2}{6} = \frac{1}{3}\ ;\ \left\{\frac{1}{3}\right\}^2 = \frac{1}{9}\ ;\ 9[(x + 2/3x + 1/9 - 1/9 - 1/9)]$$

$$9[(x + 1/3)^2 - 1/9 - 1/9)] = 9[(x + 1/3)^2 - 2/9\ ] =$$

$$9[(x + 1/3)^2 - (\sqrt{2}/\sqrt{9})^2] = 9[(x + 1/3)^2 - (\sqrt{2}/3)^2].$$

Reemplazando este valor en la 2^{da} integral, sumando y restando "6" al númerador, para obtener los diferenciales.

$$-\frac{1}{6}\int\frac{6(3x-2)\,dx}{9x^2+6x-1} = -\frac{1}{6}\int\frac{18x-12\,dx}{9x^2+6x-1} = -\frac{1}{6}\int\frac{18x+6-6-12\,dx}{9x^2+6x-1} =$$

$$-\frac{1}{6}\int\frac{(18x+6)-18\,dx}{9x^2+6x-1} = -\frac{1}{6}\left\{\int\frac{18x+6\,dx}{9x^2+6x-1} - \int\frac{18\,dx}{9x^2+6x-1}\right\}.$$

$$-\frac{1}{6}\int\frac{(18x+6)dx}{9x^2+6x-1} + \frac{18}{6}\int\frac{dx}{9[(x+1/3)^2-(\sqrt{2/3})^2]}.$$

$\left\{\begin{array}{l}\underline{\text{La } 1^{ra} \text{ integral}}\text{ , esta completa } .v = 9x^2+6x-1 \;;\; dv = 18x+6 \;; \\ \text{Se aplica} : \int\frac{dv}{v} = \ln v + c\,.\end{array}\right.$

$\left\{\begin{array}{l}\underline{\text{La } 2^{da} \text{ integral}}\text{ , tambien esta completa, } v = x+1/3 \;;\; dv = dx \;; \\ a = \sqrt{2}/3\text{ . Se aplica} : \int\frac{dv}{v^2-a^2} = \frac{1}{2a}\ln\left(\frac{v-a}{v+a}\right)+c\,.\end{array}\right.$

$$-\frac{1}{6}\int\frac{18x\,dx}{9x^2+6x-1} + \frac{3}{9}\int\frac{dx}{[(x+1/3)^2-(\sqrt{2/3})^2]} =$$

$$-\frac{1}{6}\ln\{9x^2+6x-1\} + \frac{1}{3}\cdot\frac{1}{2.\frac{\sqrt{2}}{3}}.\ln\left[\frac{x+\frac{1}{3}-\frac{\sqrt{2}}{3}}{x+\frac{1}{3}+\frac{\sqrt{2}}{3}}\right] =$$

$$-\frac{1}{6}\ln\{9x^2+6x-1\} + \frac{3}{6.\sqrt{2}}.\ln\left(\frac{\frac{3x+1-\sqrt{2}}{\cancel{3}}}{\frac{3x+1+\sqrt{2}}{\cancel{3}}}\right) =$$

$$-\frac{1}{6}\ln\{9x^2+6x-1\} + \left(\frac{1}{2\sqrt{2}}\right)\ln\left(\frac{3x+1-\sqrt{2}}{3x+1-\sqrt{2}}\right) =$$

$$-\frac{1}{6}\ln\{9x^2+6x-1\} + \frac{\sqrt{2}}{2\sqrt{2}.\sqrt{2}}\ln\left(\frac{3x+1-\sqrt{2}}{3x+1-\sqrt{2}}\right)$$

$$-\frac{1}{6}\ln\{9x^2+6x-1\} + \frac{\sqrt{2}}{2.2}\ln\left(\frac{3x+1-\sqrt{2}}{3x+1-\sqrt{2}}\right)$$

$$\mathbf{-\frac{1}{6}\ln\{9x^2+6x-1\} + \frac{\sqrt{2}}{4}\ln\left(\frac{3x+1-\sqrt{2}}{3x+1-\sqrt{2}}\right) + c\ .}$$

13. $\displaystyle\int \frac{(x+3)\,dx}{\sqrt{x^2+2x}}\ .$

Suponiendo que: $v = x^2 + 2x$; $dv = 2x + 2$.(verdadero diferencial).
□ $(x + 3)$ lo multiplicamos por 2: $2(x + 3)dx = (2x + 6)\,dx$.

$$\frac{1}{2}.\int\frac{2(x+3)\,dx}{\sqrt{x^2+2x}} = \frac{1}{2}.\int\frac{2x+6\,dx}{\sqrt{x^2+2x}} = \frac{1}{2}.\int\frac{2x+2+4\,dx}{\sqrt{x^2+2x}} =$$

$$= \frac{1}{2}.\left\{\int\frac{(2x+2)+4\,dx}{\sqrt{x^2+2x}}\right\} = \frac{1}{2}\left\{\int\frac{(2x+2)}{\sqrt{x^2+2x}} + 4\int\frac{dx}{\sqrt{x^2+2x}}\right\} =$$

$$\frac{1}{2}\int\left\{\frac{(2x+2)}{(x^2+2x)^{1/2}} + 4\int\frac{dx}{\sqrt{(x^2+2x}}\right\} =$$

Descomponiendo la cantidad sub-radical: $x^2 + 2x$

$x^2 + 2x$. $2/2 = 1$; $1^2 = 1$.

$(x + 2x + 1 - 1) = (x + 1)^2 - 1^2$. Se sustituye en la 2^da^ integral .

$$\frac{1}{2}\left\{\int\frac{(2x+2)}{(x^2+2x)^{1/2}} + 4\int\frac{dx}{\sqrt{(x+1)^2-1^2}}\right\} =$$

$$\underline{1}\int(x^2+2x)^{-1/2}.(2x+2)\,dx + \underline{1}\,.\,4\,.\int\underline{\quad dx\quad}.$$

$$2 \qquad\qquad\qquad\qquad 2 \qquad \sqrt{(x+1)^2 - 1^2}$$

$$\frac{1}{2}\int (x^2+2x)^{-1/2}.(2x+2)\,dx \;+\; \frac{1}{\cancel{2}}.\cancel{4}.\int \frac{dx}{\sqrt{(x+1)^2 - 1^2}}.$$

$$\frac{1}{2}\int (x^2+2x)^{-1/2}.(2x+2)\,dx \;+ 2\int \frac{dx}{\sqrt{(x+1)^2 - 1^2}}.$$

$\Big\{$ La 1^{ra} integral, esta completa: $v = x^2 + 2x$; $dv = 2x + 2$;
Se aplica : $\int dv/v = \ln v + c$.

$\Big\{$ La 2^{da} integral, tambien esta completa: $v = x + 1$; $dv = dx$; $a = a$.
Se aplica : $\int \frac{dv}{\sqrt{v^2 - a^2}} = \ln\left(v + \sqrt{v^2 - a^2}\right) + c$.

$$\frac{1}{2}.\frac{(x^2+2x)^{-1/2+1}}{-1/2+1} \;+\; 2\ln\left\{(x+1) + \sqrt{(x+1)^2 - 1^2}\right\} =$$

$$\frac{1}{2}.\frac{(x^2+2x)^{1/2}}{1/2} \;+\; 2\ln\left\{(x+1) + \sqrt{[(x^2+2x+1) - 1]}\right\} =$$

$$\cancel{2}.\frac{1}{\cancel{2}}.(x^2+2x)^{1/2} \;+\; 2\ln\left\{(x+1) + \sqrt{[(x^2+2x+\cancel{1} - \cancel{1}]}\right\} =$$

$$(x^2+2x)^{1/2} + 2\ln\left\{(x+1) + \sqrt{(x^2+2x)}\right\} =$$

$$\mathbf{\sqrt{(x^2+2x)} + 2\ln\{x + 1 + \sqrt{(x^2+2x)}\} + c .}$$

14. $\displaystyle \int \frac{(x+2)\,dx}{\sqrt{4x - x^2}}$.

$v = 4x - x^2$; $dv = -2x + 4$.(verdadero diferencial)

Se multiplica por (-2) al diferencial (x + 2): $-2(x+2) = -2x - 4$.
$-2(x+2) = -2x - 4$. □ se suma y resta "4" al "dv" propuesto.

$\{-2x-4\}$; $-2x-4+4-4 = -2x+4-8 = (-2x+4)-8$.

$$\frac{(-1)}{2}\left\{\int \frac{(-2)(x+2)\,dx}{\sqrt{4x-x^2}}\right\} = \frac{(-1)}{2}\left\{\int \frac{(-2x-4+4-4)dx}{\sqrt{4x-x^2}}\right\} =$$

$\frac{(-1)}{2}\int \frac{\{(-2x+4)-8\}}{\sqrt{4x-x^2}}\,dx.$ Descomponiendo: $4x - x^2 = -(x^2-4x)$.

$-(x^2-4x)$. $4/2 = 2$; $2^2 = 4$. $\{-(x^2-4x+4-4)\} =$

$\{-(x-2)^2-4)\} = \{-(x-2)^2-2^2)\} = 2^2-(x-2)^2$

$$\frac{(-1)}{2}\int \frac{\{(-2x+4)-8\}}{\sqrt{4x-x^2}}.dx = \frac{(-1)}{2}\int \frac{(-2x+4)}{\sqrt{4x-x^2}}.dx - 8\int \frac{dx}{\sqrt{2^2-(x-2)^2}} =$$

$$\frac{(-1)}{2}\int (4x-x^2)^{-1/2}.(-2x+4).dx + \frac{1}{2}.8\int \frac{dx}{\sqrt{2^2-(x-2)^2}} =$$

La 1^{ra} integral, esta completa. $v = 4x - x^2$; $dv = -2x+4$;
Se aplica : $\int v^n . dv = \frac{v^{n+1}}{n+1} + c$.

La 2^{da} integral, tambien esta completa. $v = x - 2$; $dv = dx$; $a = 2$.
Se aplica : $\int \frac{dv}{\sqrt{v^2-a^2}} =$ arc sen $\frac{v}{a} + c$.

$$-\frac{1}{2}.\frac{(4x-x^2)^{-1/2+1}}{(-1/2+1)} + \frac{8}{2}.\text{arc sen}\,\frac{x-2}{2} =$$

$$-\frac{1}{2}.\frac{(4x-x^2)^{1/2}}{(1/2)} + 4\,\text{arc sen}\,\frac{x-2}{2} + c.$$

$2/2.(4x-x^2)^{1/2} + 4\text{arc sen}\,(x-2) = \boldsymbol{\sqrt{(4x-x^2)} + 4\text{arc sen}\,(x-2) + c}$

15. $\displaystyle\int \frac{x\,dx}{\sqrt{27+6x-x^2}}$.

Multiplico por (- 2) , luego sumo y resto 6 al númerador.

$$\frac{-1}{2}\left\{\int \frac{(-2)\,x\,dx}{\sqrt{27+6x-x^2}}\right\} = \frac{-1}{2}\left\{\int \frac{(-2\,x+6-6)\,dx}{\sqrt{27+6x-x^2}}\right\} =$$

$$\frac{-1}{2}\left\{\int \frac{(-2\,x+6)-6)\,dx}{\sqrt{27+6x-x^2}}\right\} = \frac{-1}{2}\int \frac{(-2\,x+6\,)\,dx}{\sqrt{27+6x-x^2}} + \frac{1}{2}\int \frac{6dx}{\sqrt{27+6x-x^2}}$$

Descomponiendo la cantidad sub-radical del denominador de la 2da integral : $\sqrt{27+6x-x^2}$.

$27+6x-x^2 = -(x^2-6x-27)$. $6/2 = 3$; $3^2 = 9$.

$(x^2-6x-27) = -(x^2-6x+9-9-27) = -[(x-3)^2-36] =$

$-\ [(x-3)^2-6^2] = 6^2-(x-3)^2$.

Se sustituye este valor en el denominador de la 2da integral.

$$\frac{-1}{2}\left(\int (27+6x-x^2)^{-1/2}.\ (-2\,x+6\,)\,dx\right) + \frac{1}{2}\left(\int \frac{6\ dx}{\sqrt{6^2-(x-3)^2}}.\right)$$

$$\frac{-1}{2}\left(\int (27+6x-x^2)^{-1/2}.\ (-2\,x+6\,)\,dx\right) + \frac{6}{2}\left(\int \frac{dx}{\sqrt{6^2-(x-3)^2}}\right).$$

$v = 27+6x-x^2$; $dv = -2x+6\ .\ dx$; $n = -1/2$. } 1ra integral. Esta completo el diferencial. Se aplica: $\displaystyle\int v^n\,dv = \frac{v^{n+1}}{n+1} + c$.

$v = x-3$; $dv = dx$ } 2da integral: Esta completo el difererencial. Se aplica: $\displaystyle\int \frac{dv}{\sqrt{a^2-v^2}} = \text{arc sen}\ \frac{v}{a} + c.$

$$\frac{-1}{2}\left(\frac{(27+6x-x^2)^{-1/2+1}}{-1/2+1}\right)+3\ \text{arc sen}\ \frac{(x-3)}{6}+c\ .$$

$$\frac{-1}{\cancel{2}}\cdot\frac{(27+6x-x^2)^{1/2}}{(1/\cancel{2})}+3\text{arc sen}\ \frac{(x-3)}{6}+c\ .$$

$$-(27+6x-x^2)^{1/2}+3\ \text{arc sen}\ \frac{(x-3)}{6}+c\ .$$

$$\mathbf{-\sqrt{(27+6x-x^2)}+3\ \text{arc sen}\ \frac{(x-3)}{6}+c\ .}$$

16. $\displaystyle\int\frac{(3x+2)\,dx}{\sqrt{19-5x+x^2}}\ .$

Multiplico y divido para (2); luego sumo y resto (19) .

$$\frac{1}{2}\left\{\int\frac{2(3x+2)\,dx}{\sqrt{19-5x+x^2}}\right\}\ .$$

$$\frac{1}{2}\left\{\int\frac{(6x+4)\,dx}{\sqrt{19-5x+x^2}}\right\}=\frac{1}{2}\left\{\int\frac{(6x+4+19-19)dx}{\sqrt{19-5x+x^2}}\right\}=$$

$$\frac{1}{2}\left\{\int\frac{\{(6x+4-19)+19\}dx}{\sqrt{19-5x+x^2}}\right\}=\frac{1}{2}\left\{\int\frac{\{(6x-15)+19\}dx}{\sqrt{19-5x+x^2}}\right\}=$$

$$=\frac{1}{2}\left\{\int\frac{(6x-15)\,dx}{\sqrt{19-5x+x^2}}\right\}+\frac{1}{2}.19.\int\frac{dx}{\sqrt{19-5x+x^2}}\ .$$

$$\frac{1}{2}\left(\int\frac{3(2x-5)\,dx}{\sqrt{19-5x+x^2}}\right)+\frac{19}{2}\left(\int\frac{dx}{\sqrt{19-5x+x^2}}\right)=$$

$$\frac{3}{}\left(\int\frac{(2x-5)\,dx}{}\right)+\frac{19}{}\left(\int\frac{dx}{}\right).$$

$$2 \quad \sqrt{19 - 5x + x^2} \qquad 2 \quad \sqrt{19 - 5x + x^2}$$

La 1ra integral esta lista para integrarse,el radical sube como exponente negativo ; en la 2da integral primero completamos con cuadrados la cantidad sub-radical :$19 - 5x + x^2$.

$19 - 5x + x^2 = x^2 - 5x + 19$. $\quad 5/2 = 5/2$; $(5/2)^2 = 25/4$.

$$x^2 - 5x + 19 = x^2 - 5x + 19 + 25/4 - 25/4 =$$

$$(x^2 - 5x + 25/4 + 19 - 25/4) = \{(x - 5/2)^2 + 76/4 - 25/4\} =$$

$$\{(x - 5/2)^2 + 51/4\} = \{(x - 5/2)^2 + (\sqrt{51}/2)^2\}$$

Sustituyendo: $\{(x - 5/2)^2 + (\sqrt{51}/2)^2\}$ en la 2da integral .

$$\frac{3}{2}\int \frac{(2x - 5)\,dx}{\sqrt{19 - 5x + x^2}} + \frac{19}{2}\int \frac{dx}{\sqrt{19 - 5x + x^2}} .$$

$$\frac{3}{2}\int (19 - 5x + x^2)^{-1/2}.(2x - 5)\,dx + \frac{19}{2}\int \frac{dx}{\{(x - 5/2)^2 + (\sqrt{51}/2)^2\}} .$$

$v = (19 - 5x + x^2)$, $dv = 2x - 5$, $n = -1/2$ } 1ra integral. Esta completo el diferencial. Se aplica: $\int v^n\,dv = \frac{v^{n+1}}{n + 1} + c$.

$v = x - 5/2$, $dv = dx$, $a = \sqrt{51}/2$ } 2da integral. Esta completo el diferencial. Se aplica: $\int \frac{dv}{\sqrt{v^2 + a^2}} = \ln(v + \sqrt{v^2 + a^2}) + c$.

$$\frac{3}{2}\left(\frac{(19 - 5x + x^2)^{-1/2+1}}{-1/2 + 1}\right) + \frac{19}{2} \ln (x - 5/2 + \sqrt{19 - 5x + x^2}) + c .$$

$$3\left(\frac{(19 - 5x + x^2)^{1/2}}{}\right) + \frac{19}{} \ln (x - 5/2 + \sqrt{19 - 5x + x^2}) + c .$$

2 1/2 2

$$\frac{3}{\cancel{2}} \cdot \cancel{2} \cdot \sqrt{19 - 5x + x^2} + \frac{19}{2} \ln\left(x - 5/2 + \sqrt{19 - 5x + x^2}\right) + c .$$

$$\mathbf{3\sqrt{19 - 5x + x^2} + \frac{19}{2} \ln\left(x - 5/2 + \sqrt{19 - 5x + x^2}\right) + c .}$$

17. $$\int \frac{(3x - 2)\,dx}{\sqrt{4x^2 - 4x + 5}} .$$

Multiplico y divido para (8) y luego descompongo (-16) en (- 12) y (- 4)

$$\frac{1}{8}\int \frac{8(3x - 2)\,dx}{\sqrt{4x^2 - 4x + 5}} = \frac{1}{8}\int \frac{(24x - 16)\,dx}{\sqrt{4x^2 - 4x + 5}} = \frac{1}{8}\int \frac{(24x - 12 - 4)\,dx}{\sqrt{4x^2 - 4x + 5}}$$

Agrupando términos:

$$\frac{1}{8}\left(\int \frac{\{(24x - 12) - 4\}dx}{\sqrt{4x^2 - 4x + 5}}\right) = \frac{1}{8}\left(\int \frac{(24x - 12)\,dx}{\sqrt{4x^2 - 4x + 5}} - 4\int \frac{dx}{\sqrt{4x^2 - 4x + 5}}\right)$$

$$\frac{1}{8} \cdot \int \frac{3(8x - 4)\,dx}{\sqrt{4x^2 - 4x + 5}} - \frac{1}{8} \cdot 4 \cdot \int \frac{dx}{\sqrt{4x^2 - 4x + 5}} .$$

$$\frac{3}{8}\int (4x^2 - 4x + 5)^{-1/2} \cdot (8x - 4)\,dx - \frac{1}{2}\int \frac{dx}{\sqrt{4x^2 - 4x + 5}} .$$

La <u>1ra integral</u> esta lista para integrarse; la <u>2da integral</u> primero

completamos con cuadrados la cantidad sub-radical: $4x^2 - 4x + 5$

$4x^2 - 4x + 5 = 4(x^2 - x + 5/4)$. $1/2 = 1/2$; $(1/2)^2 = 1/4$.

$4(x^2 - x + 1/4 - 1/4 + 5/4) = 4(x^2 - x + 1/4 + 4/4) =$

$4\{(x - 1/2)^2 + 1\} = 4\{(x - \frac{1}{2})^2 + 1^2\} = 4\{(\frac{2x - 1}{2^2})^2 + 1^2\} =$

$\cancel{4}\{\frac{(2x - 1)^2 + 2^2}{\cancel{4}}\} = (2x - 1)^2 + 2^2$. Sustituyendo en la 2^{da} integral

Sustituyendo: $\{(2x - 1)^2 + 2^2\}$ en la 2^{da} integral .

$$\frac{3}{8}\int (4x^2 - 4x + 5)^{-1/2} . (8x - 4)\, dx - \frac{1}{2}\int \frac{dx}{(2x - 1)^2 + 2^2}.$$

$\left.\begin{array}{l} v = 2x - 1 \\ dv = 2\, dx \\ a = 2 \end{array}\right\}$ 2^{da} integral .Falta (2) para completar el diferencial. Se aplica: $\int \frac{dv}{v^2 - a^2}$.

Para la 1^{ra} integral aplicamos: $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$.

$$\frac{3}{8}\int (4x^2 - 4x + 5)^{-1/2} . (8x - 4)\, dx - \frac{1}{2}.\frac{1}{2}.\int \frac{(2)\, dx}{(2x - 1)^2 + 2^2}.$$

$$\frac{3}{8}\int (4x^2 - 4x + 5)^{-1/2} . (8x - 4)\, dx - \frac{1}{4}\int \frac{dx}{(2x - 1)^2 + 2}.$$

$$\frac{3}{8} . \frac{(4x^2 - 4x + 5)^{-1/2+1}}{-1/2 + 1} - \frac{1}{4} \ln (2x - 1 + \sqrt{4x^2 - 4x + 5}\,) + c .$$

$$\frac{3}{8} . \frac{(4x^2 - 4x + 5)^{1/2}}{1/2} - \frac{1}{4} \ln (2x - 1 + \sqrt{4x^2 - 4x + 5}\,) + c .$$

$$\frac{3}{\cancel{8}} . \cancel{2} . \sqrt{4x^2 - 4x + 5} - \frac{1}{4} \ln (2x - 1 + \sqrt{4x^2 - 4x + 5}\,) + c .$$

$$\frac{3}{4}\sqrt{(4x^2 - 4x + 5)} - \frac{1}{4}\ln\left(2x - 1 + \sqrt{4x^2 - 4x + 5}\right) + c .$$

18. $\int \frac{(8x - 3)\,dx}{\sqrt{12x - 4x^2 - 5}}$.

Haciendo artificios: $\int \frac{(8x - 12 + 9)\,dx}{\sqrt{12x - 4x^2 - 5}} = \int \frac{\{(8x - 12) + 9\}\,dx}{\sqrt{12x - 4x^2 - 5}} =$

$$\int \frac{(8x - 12)\,dx}{\sqrt{12x - 4x^2 - 5}} + \int \frac{9dx}{\sqrt{12x - 4x^2 - 5}} =$$

$$\int \frac{(8x - 12)\,dx}{\sqrt{12x - 4x^2 - 5}} + 9\int \frac{dx}{\sqrt{12x - 4x^2 - 5}} =$$

Factorizando en la 1ra integral el signo negativo:

$$\int \frac{-(- 8x + 12)\,dx}{\sqrt{12x - 4x^2 - 5}} + 9\int \frac{dx}{\sqrt{12x - 4x^2 - 5}} =$$

$$-\int \frac{(- 8x + 12)\,dx}{\sqrt{12x - 4x^2 - 5}} + 9\int \frac{dx}{\sqrt{12x - 4x^2 - 5}} .$$

$$-\int (12x - 4x^2 - 5)^{-1/2} .(- 8x + 12)\,dx + 9\int \frac{dx}{\sqrt{12x - 4x^2 - 5}} .$$

Aplicamos en la 1ra integral: $\int v^n\,dv = \frac{v^{n+1}}{n+1} + c$.

En la 2da integral:
Completamos con cuadrados la cantidad sub-radical: $12x - 4x^2 - 5$.

$12x - 4x^2 - 5 = -4(x^2 - 3x + 5/4)$. $3/2 = 3/2$; $(3/2)^2 = 9/4$.

$\{-4(x^2 - 3x + 5/4 + 9/4 - 9/4)\} = \{-4(x^2 - 3x + 9/4 + 5/4 - 9/4)\}$.

$\{-4[(x - 3/2)^2 - 4/4]\} = \{-4[(x - \frac{3}{2})^2 - 1^2]\} = \{-4[(\frac{2x - 3}{2})^2 - 1^2]\}$

$\{-4[\frac{(2x - 3)^2}{2^2} - 1]\} = \{-4[\frac{(2x - 3)^2 - 4}{4}]\} = \{\cancel{-4}[\frac{(2x - 3)^2 - 4}{\cancel{4}}]\}$

$\{-[(2x - 3)^2 - 4]\} = \{-(2x - 3)^2 + 4\} = \{4 - (2x - 3)^2\} =$

$\{2^2 - (2x - 3)^2\}$.Sustituyendo: $\{2^2 - (2x - 3)^2\}$ en la $2^{\underline{da}}$ integral .

$-\int (12x - 4x^2 - 5)^{-1/2} .(-8x + 12)\,dx + 9\int \frac{dx}{\sqrt{\{2^2 - (2x - 3)^2\}}}$.

$\left.\begin{array}{l} v = 2x - 3 \\ dv = 2\,dx \\ a = 2 \end{array}\right\}$ $2^{\underline{da}}$ integral.Falta (2) para completar el diferencial.
Se aplica: $\int \frac{dv}{\sqrt{v^2 - a^2}} = \ln\{v + \sqrt{v^2 - a^2}\} + c$.

$-\int (12x - 4x^2 - 5)^{-1/2} . (-8x + 12)\,dx + 9 . \frac{1}{2} . \int \frac{(2)dx}{\sqrt{\{(2x - 3)^2 - 2^2\}}}$.

$-\int (12x - 4x^2 - 5)^{-1/2} . (-8x + 12)\,dx + \frac{9}{2} \int \frac{(2)dx}{\sqrt{(2x - 3)^2 - 2^2}}$.

Para la $2^{\underline{da}}$ integral aplicamos: $\int \frac{dv}{\sqrt{a^2 - v^2}} = \text{arc sen } \frac{v}{a} + c$.

$\frac{(12x - 4x^2 - 5)^{-1/2+1}}{-1/2 + 1} + \frac{9}{2} \text{ arc sen } \frac{2x - 3}{2} + c$.

$\frac{(12x - 4x^2 - 5)^{1/2}}{1/2} + \frac{9}{2} \text{arc sen } \frac{2x - 3}{2} = 2(12x - 4x^2 - 5)^{1/2} + \frac{9}{2} \text{ arc sen } \frac{2x - 3}{2}$

$2\sqrt{12x - 4x^2 - 5} + \frac{9}{2} \text{ arc sen } \frac{2x - 3}{2} + c$.

Problemas - Página 256

Verificar las siguientes integraciones:

1. $\int \sqrt{1 - 4x^2}\, dx = \frac{x}{2} \sqrt{1 - 4x^2} + \frac{1}{4} \text{ arc sen } 2x + c .$

$\int \sqrt{1^2 - (2x)^2}\, dx =$

$v = 2x$ Falta (2) para completar el diferencial. Se aplica:

$dv = 2\, dx$ $\int \sqrt{a^2 - v^2}\, dv = \frac{v}{2} \sqrt{a^2 - v^2} + \frac{a^2}{2} \text{arc sen } \frac{v}{a} + c .$

$a = 1$

$$\frac{1}{2} \cdot \left\{ \int \sqrt{1 - 4x^2} \cdot (2) dx \right\} = \frac{1}{2} \cdot \left\{ \int \sqrt{1^2 - (2x)^2} \cdot (2)\, dx \right\} =$$

$$\frac{1}{2} \cdot \left\{ \frac{\cancel{2}x}{\cancel{2}} \sqrt{1 - 4x^2} + \frac{1^2}{2} \text{arc sen } \frac{2x}{1} \right\}$$

$$\frac{1}{2} \cdot \left\{ x \sqrt{1 - 4x^2} + \frac{1}{2} \text{arc sen } 2x + c \right\}$$

$$\frac{x}{2} \cdot \sqrt{1 - 4x^2} + \frac{1}{4} \cdot \text{arc sen } 2x + c .$$

2. $\int \sqrt{1 + 9x^2}\, dx = \frac{x}{2} \cdot \sqrt{1 - 9x^2} + \frac{1}{6} \ln (3x + \sqrt{1 - 9x^2}) + c .$

$\int \sqrt{1^2 + (3x)^2}\, dx =$

$v = 3x$ Falta (3) para completar el diferencial, se aplica:
$dv = 3\,dx$ $\int \sqrt{a^2 + v^2}\,dv = \frac{v}{2}\sqrt{v^2 + a^2} + \frac{a^2}{2}\ln\left(v + \sqrt{v^2 + a^2}\right) + .$
$a = 1$

$$\frac{1}{3}\left\{\int \sqrt{1^2 + (3x)^2}.(3)\,.dx\right\} =$$

$$\frac{1}{3}\left\{\frac{3x}{2}.\sqrt{1 + 9x^2} + \frac{1^2}{2}\ln\left(3x + \sqrt{1 + 9x^2}\right)\right\} + c$$

$$\frac{\cancel{1}}{\cancel{3}}\left\{\frac{\cancel{3}x}{2}.\sqrt{1 + 9x^2}\right\} + \frac{1}{3}\left\{\frac{1}{2}\ln\left(3x + \sqrt{1 + 9x^2}\right)\right\} + c$$

$$\mathbf{\frac{x}{2}.\sqrt{1 + 9x^2} + \frac{1}{6}\ln\left[3x + \sqrt{1 + 9x^2}\right] + c\,.}$$

3. $\int \sqrt{\frac{x^2 - 1}{4}}\,dx = x\sqrt{x^2 - 4} - \ln\left(x + \sqrt{x^2 - 4}\right) + c\,.$

$$\int \sqrt{\frac{x^2 - 4}{4}}\,dx = \int \frac{\sqrt{x^2 - 4}}{\sqrt{4}}\,dx = \frac{1}{2}.\int \sqrt{x^2 - 4}\,dx = \frac{1}{2}.\int \sqrt{x^2 - 2}$$
dx

$v = x$ El diferencial esta completo. Se aplica:
$dv = dx$ $\int \sqrt{v^2 - a^2}\,dv = \frac{v}{2}\sqrt{v^2 - a^2} - \frac{a^2}{2}\ln\left(v + \sqrt{v^2 - a^2}\right) + c\,.$
$a = 2$

$$\frac{1}{2}.\left\{\frac{x}{2}\sqrt{x^2 - 4} - \frac{2^2}{2}.\ln\left(x + \sqrt{x^2 - 4}\right) + c\right\}$$

$$\frac{1}{2}.\left\{\frac{x}{2}\sqrt{x^2 - 4}\right\} - \left\{\frac{1}{2}.\frac{4}{2}.\ln\left(x + \sqrt{x^2 - 4}\right)\right\} + c\,.$$

$\frac{x}{4}\sqrt{x^2-4} - .\frac{\cancel{4}}{\cancel{4}}.\ln(x+\sqrt{x^2-4}) + c .$

$\mathbf{\frac{x}{4}\sqrt{x^2-4} - \ln(x+\sqrt{x^2-4}) + c .}$

4. $\mathbf{\int \sqrt{25-9x^2}\ dx = \frac{x}{2}\sqrt{25-9x^2} + \frac{25}{6}\ \text{arc sen}\ \frac{3x}{5} + c .}$

$\int \sqrt{25-9x^2}\ dx = \int \sqrt{5^2-(3x)^2}\ dx =$

$$\left.\begin{array}{l} v = 3x \\ dv = 3\ dx \\ a = 5 \end{array}\right\} \begin{array}{l}\text{Falta (3) para completar el diferencial. Se aplica:} \\ \int \sqrt{a^2-v^2}\ dv = \frac{v}{2}\sqrt{a^2-v^2} + \frac{a^2}{2}\ \text{arc sen}\ \frac{v}{a} + c . \end{array}$$

$\int \sqrt{5^2-(3x)^2}\ dx = \frac{1}{3}\left\{ \int \sqrt{5^2-(3x)^2}\ .(3)dx \right\}$

$= \frac{1}{3}\left\{ \frac{3x}{2}.\sqrt{25-9x^2} + \frac{5^2}{2}\ \text{arc sen}\ \frac{3x}{5} \right\} + c .$

$= \frac{1}{\cancel{3}}.\frac{\cancel{3}x}{2}.\sqrt{25-9x^2} + \frac{1}{3}.\frac{5^2}{2}\ \text{arc sen}\ \frac{3x}{5} + c .$

$\mathbf{= \frac{x}{2}.\sqrt{25-9x^2} + \frac{25}{6}\ \text{arc sen}\ \frac{3x}{5} + c .}$

5. $\mathbf{\int \sqrt{4x^2+9}\ dx = \frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}\ln(2x+\sqrt{4x^2+9}) + c .}$

$\int \sqrt{4x^2+9}\ dx = \sqrt{(2x)^2+3^2}\ dx .$

$$\left.\begin{array}{l} v = 2x \\ dv = 2\ dx \\ \ \end{array}\right\} \begin{array}{l}\text{Falta (2) para completar el diferencial. Se aplica:} \\ \int \sqrt{a^2+v^2}\,dv = \underline{v}\ \sqrt{a^2+v^2} + \underline{a^2}\ \ln(v+\sqrt{a^2+v^2}) + c . \end{array}$$

$a = 3 \qquad 2 \qquad 2$

$$\frac{1}{2}\left\{\sqrt{(2x)^2 + 3^2}.(2)\,dx\right\} =$$

$$\frac{1}{2}\left\{\frac{2x}{2}.\sqrt{4x^2+9} + \frac{3^2}{2}\ln(2x+\sqrt{4x^2+9}) + c.\right\}$$

$$\frac{1}{2}.\frac{2x}{2}.\sqrt{4x^2+9} + \frac{1}{2}.\frac{3^2}{2}\ln(2x+\sqrt{4x^2+9}) + c.$$

$$\frac{1}{2}.\frac{\cancel{2}x}{\cancel{2}}.\sqrt{4x^2+9} + \frac{9}{4}\ln(2x+\sqrt{4x^2+9}) + c.$$

$$\mathbf{\frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}\ln(2x+\sqrt{4x^2+9}) + c.}$$

6. $\displaystyle\int \sqrt{5-3x^2}\,dx = \frac{x}{2}\sqrt{5-3x^2} + \frac{5}{2\sqrt{3}}\text{ arc sen}\left(x\sqrt{\frac{3}{5}}\right) + c.$

$$\int \sqrt{5-3x^2}\,dx = \int \sqrt{(\sqrt{5})^2 - (\sqrt{3}.x)^2}\,dx.$$

$v = \sqrt{3}.x$

$dv = \sqrt{3}\,dx$

$a = \sqrt{5}$

Falta $(\sqrt{3})$ para completar el diferencial.Se aplica: $\displaystyle\int a^2 - v^2\,dv = \frac{v}{2}\sqrt{a^2-v^2} + \frac{a^2}{2}\text{ arc sen }\frac{v}{a} + c.$

$$\frac{1}{\sqrt{3}}\left\{\int \sqrt{\{(\sqrt{5})^2 - (\sqrt{3}.x)^2\}}.\sqrt{3}\,dx\right\} =$$

$$\frac{1}{\sqrt{3}}\left\{\frac{\sqrt{3}\,x}{2}.\sqrt{5-3x^2} + \frac{(\sqrt{5})^2}{2}\text{ arc sen}\left(\frac{\sqrt{3}.x}{\sqrt{5}}\right) + c\right\}$$

$$\frac{1}{\cancel{\sqrt{3}}}.\frac{\cancel{\sqrt{3}}\,x}{2}.\sqrt{5-3x^2} + \frac{1}{\sqrt{3}}.\frac{(\sqrt{5})^2}{2}\text{ arc sen}\left(\sqrt{\frac{3}{5}}.x\right) + c.$$

$$\frac{x}{2} \cdot \sqrt{5 - 3x^2} + \frac{5}{2\sqrt{3}} \cdot \text{arc sen } x \cdot \sqrt{\frac{3}{5}} + c .$$

7. $\int \sqrt{3 - 2x - x^2} \; . \; dx = \frac{x+1}{2} \sqrt{3 - 2x - x^2} + 2 \text{ arc sen } \frac{x+1}{2} + c .$

Factorizamos y completamos con cuadrados : $3 - 2x - x^2$.

$3 - 2x - x^2 = -(x^2 + 2x - 3)$. $2/2 = 1$; $1^2 = 1$

$-(x^2 + 2x + 1 - 1 - 3) = -\{(x + 1)^2 - 4\} = -\{(x + 1)^2 - 2^2\} =$

$2^2 - (x + 1)^2$; reemplazando este resultado en la integral .

$\int \sqrt{3 - 2x - x^2} \; .dx = \int \sqrt{2^2 - (x + 1)^2} \; .dx =$

$\left.\begin{array}{l} v = x + 1 \\ dv = dx \\ a = 2 \end{array}\right\}$ El diferencial esta completo. Se aplica: $\int \sqrt{a^2 - v^2} \; . \; dv = \frac{v}{2} \sqrt{a^2 - v^2} + \frac{a^2}{2} \text{ arc sen } \frac{v}{a} + c .$

$\int \sqrt{22 - (x + 1)^2} \; .dx = \frac{x+1}{2} \sqrt{3 - 2x - x^2} + \frac{2^2}{2} \text{ arc sen } \left(\frac{x+1}{2}\right) + c$

$\frac{x+1}{2} \cdot \sqrt{3 - 2x - x^2} + \frac{2^2}{2} \text{ arc sen } \left(\frac{x+1}{2}\right) + c .$

$\frac{x+1}{2} \sqrt{3 - 2x - x^2} + \frac{4}{2} \text{ arc sen } \left(\frac{x+1}{2}\right) + c .$

$$\frac{x+1}{2} \sqrt{3 - 2x - x^2} + 2 \text{ arc sen } \frac{x+1}{2} + c .$$

8. $\int \sqrt{5 - 2x + x^2} \, .dx = \frac{x - 1}{2} \sqrt{5 - 2x + x^2} + 2\ln (x - 1 + 5 - 2x + x^2) + c$

Factorizamos y completamos con cuadrados: $5 - 2x + x^2$.

$5 - 2x + x^2 = (x^2 - 2x + 5)$. $2/2 = 1$; $1^2 = 1$.

$(x^2 - 2x + 1 + 5 - 1) = \{(x - 1)^2 + 4\} = \{(x - 1)^2 + 2^2\} =$

$(x - 1)^2 + 2^2$; reemplazando este resultado en la integral .

$\int \sqrt{5 - 2x + x^2} \, . \, dx = \int \sqrt{(x - 1)^2 + 2^2} \, . \, dx$.

$\left.\begin{array}{l} v = x - 1 \\ dv = dx \\ a = 2 \end{array}\right\}$ El diferencial esta completo. Se aplica: $\int \sqrt{v^2 + a^2} \, . \, dv = \frac{v}{2} \sqrt{v^2 + a^2} + \frac{a^2}{2} \ln (v + \sqrt{v^2 + a^2}) + c$

$\int \sqrt{(x - 1)^2 + 2^2} . dx = \frac{x - 1}{2} \sqrt{5 - 2x + x^2} + \frac{2^2}{2} \ln (x - 1 + \sqrt{5 - 2x + x^2})$

$\frac{x - 1}{2} \sqrt{5 - 2x + x^2} + \frac{4}{2} \ln (x - 1 + \sqrt{5 - 2x + x^2}) + c$.

$\mathbf{\frac{x - 1}{2} \sqrt{5 - 2x + x^2} + 2 \ln [x - 1 + \sqrt{5 - 2x + x^2}] + c}$ **.**

9. $\mathbf{\int \sqrt{2x - x^2} \, . \, dx = \frac{x - 1}{2} \sqrt{2x - x^2} + \frac{1}{2} \text{arc sen } (x - 1) + c}$ **.**

Factorizamos y completamos con cuadrados: $2x - x^2$.

$- x^2 + 2x = - (x^2 - 2x)$. $2/2 = 1$; $1^2 = 1$.

$- (x^2 - 2x) = - (x^2 - 2x + 1 - 1) = - \{(x - 1)^2 - 1\} = - \{(x - 1)^2 - 1^2\}$

$1^2 - (x - 1)^2$; reemplazando este resultado en la integral .

$\int \sqrt{2x - x^2} \, . \, dx = \int \sqrt{1^2 - (x - 1)^2} \, . \, dx$.

$\left.\begin{array}{l} v = x - 1 \\ dv = dx \\ a = 1 \end{array}\right\}$ El diferencial esta completo. Se aplica: $\int \sqrt{a^2 - v^2}.\ dv = \frac{v}{2}\sqrt{a^2 - v^2} + \frac{a^2}{2}\ \text{arc sen}\ \frac{v}{a} + c.$

$$\int \sqrt{1^2 - (x-1)^2}\ .\ dx = \frac{x-1}{2}\sqrt{2x - x^2} + \frac{1^2}{2}\ \text{arc sen}\left(\frac{x-1}{1}\right) + c.$$

$$\mathbf{\frac{x-1}{2}.\ \sqrt{2x - x^2} + \frac{1}{2}\ arc\ sen\ (x-1) + c.}$$

10. $\int \sqrt{10 - 4x + 4x^2}.\mathbf{dx} = \frac{2x-1}{4}\sqrt{10 - 4x + 4x^2} + \frac{9}{4}\ \ln\ (2x - 1 + \sqrt{10 - 4x + 4x^2}) + c$

$$4x^2 - 4x + 10 = 4(x^2 - x + 10/4\ ) = 4(x^2 - x + 10/4\ )\ .$$

$$1/2 = 1/2\ \ ;\ \ (1/2)^2 = 1/4$$

$$4(x^2 - x + 10/4) = 4(x^2 - x + 1/4 + 10/4 - 1/4)$$

$$4\left\{\left(x - \frac{1}{2}\right)^2 + \frac{9}{4}\right\} = 4\left\{\left[\frac{2x-1}{2}\right]^2 + \frac{9}{4}\right\} 4\left\{\frac{(2x-1)^2}{4} + \frac{9}{4}\right\}$$

$$\cancel{4}.\ \frac{(2x-1)^2}{\cancel{4}} + \cancel{4}.\ \frac{9}{\cancel{4}} = (2x-1)^2 + 9 = (2x-1)^2 + 3^2$$

Reemplazando este resultado en la integral .

$$\int \sqrt{(2x-1)^2 + 3^2}\ .\ dx\ .$$

$\left.\begin{array}{l} v = 2x - 1 \\ dv = 2\ .dx \\ a = 3 \end{array}\right\}$ Falta (2) para completar el diferencial.Se aplica: $\int \sqrt{v^2 + a^2}.\ dv = \frac{v}{2}\sqrt{v^2 + a^2} + \frac{a^2}{2}\ln(v + \sqrt{v^2 + a^2}) + c\ .$

$$\frac{1}{}\left\{\int \sqrt{\{(2x-1)^2 + 3^2\}}\ .\ (2)\ \ dx\right\}.$$

2

$$\frac{1}{2}\left\{\frac{(2x-1)}{2}\sqrt{10-4x+4x^2}+\frac{3^2}{2}\ln\left[(2x-1)+\sqrt{10-4x+4x^2}\right]\right\}+c\ .$$

$$\mathbf{\frac{(2x-1)}{4}\sqrt{10-4x+4x^2}+\frac{9}{4}\ln\{(2x-1)+\sqrt{10-4x+4x^2}\}+c\ .}$$

11. $\int \sqrt{16-9x^2}\ .\ dx\ .$

$$\int \sqrt{(4)^2-(3x)^2}\ .\ dx$$

$$\left.\begin{array}{l} v = 3x \\ dv = 3\ dx \\ a = 4 \end{array}\right\} \begin{array}{l}\text{Falta (3) para completar el diferencial. Se aplica:} \\ \int \sqrt{a^2-v^2}\ .\ dv = \frac{v}{2}\sqrt{a^2-v^2}+\frac{a^2}{2}\text{ arc sen }\frac{v}{a}+c\ .\end{array}$$

$$\frac{1}{3}.\int \sqrt{(4)^2-(3x)^2}\ .(3)\ dx = \frac{3x}{2}\sqrt{16-9x^2}+\frac{4^2}{2}\text{ arc sen }\left(\frac{3x}{4}\right)+c\ .$$

$$\frac{3x}{2}\sqrt{16-9x^2}+\frac{16}{2}\text{ arc sen }\frac{3x}{4} = \mathbf{\left(\frac{3x}{2}\right)\sqrt{16-9x^2}+8\text{ arc sen }\frac{3x}{4}+c}$$

12. $\int \sqrt{4+25x^2}\ .\ dx\ .$

$$\int \sqrt{2^2+(5x)^2}\ .\ dx =$$

$$\left.\begin{array}{l} v = 5x \\ dv = 5\ dx \\ a = 2 \end{array}\right\} \begin{array}{l}\text{Falta (5) para completar el diferencial. Se aplica:} \\ \int \sqrt{a^2-v^2}\ .\ dv = \frac{v}{2}\sqrt{a^2-v^2}+\frac{a^2}{2}\text{ arc sen }\frac{v}{a}+c\ .\end{array}$$

$$\int \sqrt{2^2+(5x)^2}\ .\ dx = \frac{5x}{2}\sqrt{4+25x^2}+\frac{2^2}{2}\text{ arc sen }\frac{3x}{2}+c$$

$$\frac{5x}{2}\sqrt{4+25x^2} + 2 \text{ arc sen } \frac{3x}{2} + c .$$

13. $\int \sqrt{9x^2 - 1}\ dx .$

$\int \sqrt{(3x)^2 - 1^2} . dx =$

$v = 3x$
$dv = 3\ dx$
$a = 1$

Falta (3) para completar el diferencial. Se aplica: $\int \sqrt{v^2 - a^2} . dv = \frac{v}{2}\sqrt{v^2 - a^2} - \frac{a^2}{2}\ln(v + \sqrt{v^2 - a^2}) + c .$

$$\int \sqrt{(3x)^2 - 1^2} . dx = \frac{3x}{2}\sqrt{9x^2 - 1} - \frac{1^2}{2}\ln(3x + \sqrt{9x^2 - 1}) + c .$$

$$\frac{3x}{2}\sqrt{9x^2 - 1} - \frac{1}{2}\ln(3x + \sqrt{9x^2 - 1}) + c .$$

14. $\int \sqrt{8 - 3x^2} . dx .$

$\int \sqrt{(\sqrt{8})^2 - (\sqrt{3} . x)^2} . dx =$

$v = \sqrt{3} . x$
$dv = \sqrt{3}\ dx$
$a = \sqrt{8}$

Falta ($\sqrt{3}$) para completar el diferencial. Se aplica: $\int \sqrt{a^2 - v^2} . dv = \frac{v}{2}\sqrt{a^2 - v^2} - \frac{a^2}{2} \text{ arc sen } \frac{v}{a} + c$

$$\frac{1}{\sqrt{3}} . \left\{\int \sqrt{(\sqrt{8})^2 - (\sqrt{3} . x)^2} . \sqrt{3}\ dx\right\} =$$

$$\frac{1}{\sqrt{3}}\left\{\frac{\sqrt{3}.x}{2} . \sqrt{(\sqrt{8})^2 - (\sqrt{3} . x)^2} - \frac{(\sqrt{8})^2}{2} \text{ arc sen } \left(\frac{\sqrt{3}.x}{\sqrt{8}}\right)\right\} + c .$$

$$\frac{1}{\cancel{\sqrt{3}}} . \frac{\cancel{\sqrt{3}}.x}{2}\sqrt{8 - 3x^2} - \frac{1}{\sqrt{3}} . \frac{8}{2} \text{ arc sen } \left(\frac{\sqrt{3}.x.\sqrt{8}}{\sqrt{8}.\sqrt{8}}\right) + c .$$

$$\frac{x}{}\sqrt{8 - 3x^2} - \frac{4.\sqrt{3}}{} \text{ arc sen } \left(\frac{\sqrt{24}.x}{}\right) + c .$$

$2 \qquad \sqrt{3}.\sqrt{3} \qquad 8$

$$\frac{x}{2}\sqrt{8-3x^2} - \frac{4\sqrt{3}}{3}\text{ arc sen }\left(\frac{2x\sqrt{6}}{8}\right) + c\ .$$

15. $\int \sqrt{5+2x^2}\ .\ dx\ .$

$\int \sqrt{(\sqrt{5})^2 + (\sqrt{2}.x)^2}\ .\ dx =$

$$\left.\begin{matrix} v = \sqrt{2}\ .\ x \\ dv = \sqrt{2}\ dx \\ a = \sqrt{5} \end{matrix}\right\}$$ Falta $(\sqrt{2})$ para completar el diferencial. Se aplica; $\int \sqrt{a^2+v^2}\ .\ dv = \frac{v}{2}\sqrt{a^2+v^2} + \frac{a^2}{2}\ln\{v + \sqrt{a^2+v^2}\} + c$

$$\frac{1}{\sqrt{2}}\left\{\int \sqrt{(\sqrt{5})^2 + (\sqrt{2}.x)^2}.(\sqrt{2})\ dx\right\}$$

$$\frac{1}{\sqrt{2}}\left\{\frac{\sqrt{2}.x}{2}\ .\ \sqrt{5+2x^2} + \frac{(\sqrt{5})^2}{2}\ln\left[\sqrt{2}.x + \sqrt{5+2x^2}\right]\right\}$$

$$\left(\frac{\cancel{\sqrt{2}}.x}{2\cancel{\sqrt{2}}}\ .\ \sqrt{5+2x^2}\right) + \left(\frac{1}{\sqrt{2}}\ .\ \frac{5}{2}\ln\left[\sqrt{2}.x + \sqrt{5+2x^2}\right]\right)$$

$$\frac{x}{2}.\ \sqrt{5+2x^2} + \frac{5.\sqrt{2}}{2\sqrt{2}.\sqrt{2}}\ln\left[\sqrt{2}.x + \sqrt{5+2x^2}\right] + c\ .$$

$$\frac{x}{2}\sqrt{5+2x^2} + \frac{5\sqrt{2}}{2x2}\ln\left[\sqrt{2}.x + \sqrt{5+2x^2}\right] + c\ .$$

$$\frac{x}{2}\sqrt{5+2x^2} + \frac{5\sqrt{2}}{4}\ln\left[x\sqrt{2} + \sqrt{5+2x^2}\right] + c\ .$$

16. $\int \sqrt{5-4x-x^2}\ .\ dx\ .$

Factorizamos y completamos con cuadrados: $5 - 4x - x^2$.

$-x^2 - 4x + 5 = -(x^2 + 4x - 5)$. $4/2 = 2$; $(2)^2 = 4$

$-(x^2 + 4x - 5) = -(x^2 + 4x + 4 - 5 - 4)$

$-\{(x^2 + 4x + 4) - 9\} = -\{(x + 2)^2 - 3^2\} = 3^2 - (x + 2)^2$

Reemplazando este <u>ultimo resultado</u> en la integral .

$\int \sqrt{5 - 4x - x^2} \, . \, dx = \int \sqrt{3^2 - (x + 2)^2} . \, dx =$

$\left.\begin{array}{l} v = x + 2 \\ dv = dx \\ a = \sqrt{5} \end{array}\right\}$ El diferencial esta completo. Se aplica: $\int \sqrt{a^2 - v^2} . dv = \dfrac{v}{2} \sqrt{a^2 - v^2} + \dfrac{a^2}{2} \text{ arc sen } \dfrac{v}{a} + c$.

$\dfrac{x + 2}{2} \sqrt{5 - 4x - x^2} + \dfrac{(\sqrt{5})^2}{2} \text{ arc sen} \left(\dfrac{x + 2}{\sqrt{5}}\right) + c$.

$\dfrac{x + 2}{2} \sqrt{5 - 4x - x^2} + \dfrac{5}{2} \text{ arc sen} \left(\dfrac{(x + 2). \sqrt{5}}{\sqrt{5} . \sqrt{5}}\right) + c$.

$\left(\dfrac{x + 2}{2}\right) \sqrt{5 - 4x - x^2} + \dfrac{5}{2} \text{ arc sen } \dfrac{(x + 2). \sqrt{5}}{5} + c$.

17. $\int \sqrt{5 + 2x + x^2} \, . \, dx =$

Factorizamos y completamos con cuadrados: $5 + 2x + x^2$.
$5 + 2x + x^2 = (x^2 + 2x + 5)$. $2/2 = 1$; $(1)^2 = 1$

Sumando y restando (1) en: $(x^2 + 2x + 5) = (x^2 + 2x + 1 + 5 - 1)$

$\{(x^2 + 2x + 1) + (5 - 1)\} = \{(x + 1)^2 + 4\} = \{(x + 1)^2 + 2^2\}$

Reemplazando este resultado en la integral .

$\int \sqrt{5 + 2x + x^2} \, . \, dx = \int \sqrt{(x + 1)^2 + 2^2} \, . \, dx =$

$$\left.\begin{array}{l} v = x + 1 \\ dv = dx \\ a = 2 \end{array}\right\} \begin{array}{l} \text{El diferencial esta completo. Se aplica:} \\ \int \sqrt{v^2 + a^2}.dv = \frac{v}{2}\sqrt{v^2 + a^2} + \frac{a^2}{2}\ln(\sqrt{v^2 + a^2}\ \frac{v}{a} + c \end{array}$$

$$\frac{x + 1}{2}\sqrt{5 + 2x + x^2} + \frac{(2)^2}{2}\ln\{(x + 2) + \sqrt{5 + 2x + x^2}\} + c\ .$$

$$\frac{x + 1}{2}\sqrt{5 + 2x + x^2} + \frac{4}{2}\ln\{(x + 2) + \sqrt{5 + 2x + x^2}\} + c\ .$$

$$\mathbf{\frac{x + 1}{2}\sqrt{5 - 4x - x^2} + 2\ \ln\{(x + 2) + \sqrt{5 + 2x + x^2}\} + c\ .}$$

18. $\int \sqrt{x^2 - 8x + 7}\ .\ dx\ .$

Factorizamos y completamos con cuadrados: $x^2 - 8x + 7$.
$x^2 - 8x + 7$. $8/2 = 4$; $(4)^2 = 16$

Sumando y restando (16) en : $(x^2 - 8x + 16 + 7 - 16) =$
$\{(x^2 - 8x + 16) + (7 - 16)\} = \{(x - 4)^2 - 9\} = \{(x - 4)^2 - 3^2\}$.

Reemplazando este resultado en la integral .

$$\int \sqrt{x^2 - 8x + 7}\ .\ dx = \int \sqrt{(x - 4)^2 - 3^2}.dx =$$

$$\left.\begin{array}{l} v = x - 4 \\ dv = dx \\ a = 3 \end{array}\right\} \begin{array}{l} \text{El diferencial esta completo. Se aplica:} \\ \int \sqrt{v^2 - a^2}.dv = \frac{v}{2}\sqrt{v^2 - a^2} - \frac{a^2}{2}\ln(v + \sqrt{v^2 - a^2}) + c\ . \end{array}$$

$$\frac{x - 4}{2}\sqrt{x^2 - 8x + 7} - \frac{(3)^2}{2}\ln\{(x - 4) + \sqrt{x^2 - 8x + 7}\} + c\ .$$

$$\mathbf{\frac{x - 4}{2}\sqrt{x^2 - 8x + 7} - \frac{9}{2}\ln\{(x - 4) + \sqrt{x^2 - 8x + 7}\} + c\ .}$$

19. $\int \sqrt{4 - 2x - x^2}\ .\ dx$ **.**

Factorizamos y completamos con cuadrados: $4 - 2x - x^2$.

$-x^2 - 2x + 4 = -(x^2 + 2x - 4)$. $2/2 = 1$; $(1)^2 = 1$

Sumando y restando (1) en: $-(x^2 + 2x - 4) = -(x^2 + 2x + 1 - 4 - 1)$

$-\{(x^2 + 2x + 1) + (-4 - 1)\} = -\{(x + 1)^2 - 5\} = -\{(x + 1)^2 - (\sqrt{5})^2\}$

$(\sqrt{5})^2 - (x + 1)^2$. Reemplazando este resultado en la integral .

$\int \sqrt{4 - 2x - x^2}\ .\ dx = \int \sqrt{(\sqrt{5})^2 - (x + 1)^2}\ .\ dx =$

$$\left.\begin{array}{l} v = x + 1 \\ dv = dx \\ a = \sqrt{5} \end{array}\right\} \text{El diferencial esta completo. Se aplica: } \int \sqrt{a^2 - v^2}.dv = \frac{v}{2}\sqrt{a^2 - v^2} + \frac{a^2}{2} \text{ arc sen } \frac{v}{a} + c .$$

$$\frac{x + 1}{2}\sqrt{4 - 2x - x^2} + \frac{(\sqrt{5})^2}{2} \text{ arc sen}\left[\frac{(x + 1).\sqrt{5}}{\sqrt{5}.\sqrt{5}}\right] + c .$$

$$\left(\frac{x + 1}{2}\right)\sqrt{4 - 2x - x^2} + \frac{5}{2} \text{ arc sen}\left(\frac{(x + 1).\sqrt{5}}{5}\right) + c .$$

20. $\int \sqrt{x^2 - 2x + 8}\ .\ dx$ **.**

Factorizamos y completamos con cuadrados: $x^2 - 2x + 8$.
$x^2 - 2x + 8$. $2/2 = 1$; $(1)^2 = 1$

Sumando y restando (1) en : $x^2 - 2x + 8 = x^2 - 2x + 1 + 8 - 1 =$
$\{(x^2 - 2x + 1) + (8 - 1)\} = \{(x - 1)^2 + (7)\} = \{(x - 1)^2 + (\sqrt{7})^2\}$
Reemplazando este resultado en la integral .

$\int \sqrt{x^2 - 2x + 8}\ .\ dx = \int \sqrt{(x - 1)^2 + (\sqrt{7})^2}\ .\ dx =$

$$\left.\begin{array}{l} v = x - 1 \\ \\ \end{array}\right\} \text{El diferencial esta completo. Se aplica:}$$

$dv = dx \quad \int \sqrt{v^2 + a^2}.dv = \frac{v}{2}\sqrt{v^2 + a^2} + \frac{a^2}{2}\ln(v + \sqrt{v^2 + a^2}) + c$.

$a = \sqrt{7}$

$$\frac{x - 1}{2}\sqrt{x^2 - 2x + 8} + \frac{(3)^2}{2}\ln\{(x - 1) + \sqrt{x^2 - 2x + 8}\} + c .$$

$$\left(\frac{x - 1}{2}\right)\sqrt{x^2 - 8x + 7} + \frac{9}{2}\ln\{(x - 1) + \sqrt{x^2 - 8x + 7}\} + c .$$

$$\left(\frac{x - 1}{2}\right)\sqrt{x^2 - 2x + 8} + \frac{9}{2}\ln\{(x - 1) + \sqrt{x^2 - 2x + 8}\} + c .$$

Problemas. Páginas 259 y 260

Verificar las siguientes Integraciones:

1. $\int \operatorname{sen}^3 x\, dx = 1/3 \ \cos^3 x - \cos x + c$ **.**

Por trigonometria:

$\operatorname{sen}^2 x + \cos^2 x = 1$; $\operatorname{sen}^2 x = 1 - \cos^2 x$; $\cos^2 x = 1 - \operatorname{sen}^2 x$.

$\operatorname{sen}^3 x = \operatorname{sen}^2 x \ . \ \operatorname{sen} x$.

Sustituyendo este valor en la integral

y aplicando sustituciones trigonométricas : $\operatorname{sen}^2 x = 1 - \cos^2 x$.

$\int \operatorname{sen}^3 x\, dx = \int \operatorname{sen}^2 x \ . \ \operatorname{sen} x \ . \ dx = \int (1 - \cos^2 x) \ .\operatorname{sen} x \ . \ dx =$

$\int \operatorname{sen} x \ . \ dx \ - \ \int \cos^2 x \ . \ \operatorname{sen} x \ . \ dx$.

$v = \cos x$

$dv = - \operatorname{sen} x\, dx$

1ra integral ,esta completa, se integra.

2da integral ,le falta el signo (-) para

completar el diferencial.

$\int$ sen x . dx - (-) $\int$ $(\cos x)^2$.(-) sen x . dx .

$\int$ sen x . dx + $\int$ $(\cos x)^2$.(-) sen x . dx

$-\cos x + \dfrac{(\cos x)^{2+1}}{2+1} = -\cos x + \dfrac{(\cos x)^3}{3} = \mathbf{1/3\,(\cos x)^3 - \cos x + c}$.

2. $\int \mathbf{sen^2\,\theta\ .cos\,\theta\ .\ d\theta = 1/3\ sen^3\theta + c}$.

$\int$ $(\text{sen}\ \theta)^2$ .cos θ dθ .

$\left.\begin{array}{l} v = \text{sen}\ \theta \\ dv = \cos\theta\ d\theta \\ n = 2 \end{array}\right\}$ El diferencial esta completo, se procede a a integrar.

$\dfrac{(\cos\theta)^{2+1}}{2+1} = \dfrac{(\cos\theta)^3}{3} = \mathbf{1/3\ cos^3\theta + c}$.

3. $\int \mathbf{cos^2\varphi\quad sen\ \varphi\quad d\varphi}$.

$\int$ $(\cos\varphi\ )^2$. sen φ dφ

$\left.\begin{array}{l} v = \cos\varphi \\ dv = -\ \text{sen}\ \varphi\ \ d\varphi \\ n = 2 \end{array}\right\}$ Le falta el signo (-) , para completar el diferencial,luego se procede a integrar.

$(-)\int (\cos\varphi\ )^2.(-)\text{sen}\ \varphi\ \ d\varphi = -\dfrac{(\cos\varphi\ )^{2+1}}{2+1} = \dfrac{-\cos^3\varphi}{} = \mathbf{\dfrac{-\,1/3\ cos^3\ \varphi}{3}} + \mathbf{c}$

4. $\int \mathbf{sen^3\,6x\ .\ cos\ 6x\ \ dx}$.

$\int$ $\text{sen}^3\,6x$. cos 6x dx = $\int$ $(\text{sen}\ 6x)^3$. cos 6x dx

$v = \text{sen } 6x$ Le falta (6) para completar el diferencial,
$dv = \cos 6x \, . \, 6 \, dx$ luego se procede a integrar.
$n = 3$

$$\left(\frac{1}{6}\right)\int (\text{sen } 6x)^3 . \cos 6x \, .(6)dx = \frac{1}{6} . \frac{(\text{sen } 6x)^{3+1}}{3+1} = \frac{1}{6} . \frac{(\text{sen } 6x)^4}{4} =$$

$$\frac{(\text{sen } 6x)^4}{24} = \mathbf{\frac{1}{24} \, sen^4 \, 6x + c} \text{ .\_}$$

5. $\int \cos^3 2\theta \, . \, \text{sen } 2\theta \, . \, d\theta = - 1/8 \cos^4 2\theta + c$ **.**

$\int (\cos 2\theta)^3 . \text{ sen } 2\theta \, . \, d\theta =$

$$\left.\begin{array}{l} v = \cos 2\theta \\ dv = - \text{sen } 2\theta \, . \, 2d\theta \\ n = 3 \end{array}\right\} \text{ Falta (- 2) para completar el diferencial.}$$

$$(-\frac{1}{2})\int (\cos 2\theta)^3 . \text{sen } 2\theta \, . \, (-2) \, d\theta = -\frac{1}{2} . \frac{(\cos 2\theta)^{3+1}}{3+1} = - \frac{(\cos 2\theta)^4}{2(4)} =$$

$$- \frac{(\cos 2\theta)^4}{8} = - 1/8 \, (\cos 2\theta)^4 = \mathbf{- 1/8 \cos^4 2\theta + c} \text{ .\_}$$

6. $\int \frac{\cos^3 x}{\text{sen}^4 x} . \, dx = \csc x - 1/3 \csc^3 x + c$ **.**

$$\int (\text{sen } x)^{-4} . \cos^3 x \, dx = \int (\text{sen } x)^{-4} . \cos^2 x \, . \cos x \, dx =$$
$$\int \{(\text{sen } x)^{-4} . \cos^2 x\} . \cos x \, dx = \int \{(\text{sen } x)^{-4} (1 - \text{sen}^2 x)\} . \cos x \, dx =$$

Haciendo operaciones:

$$\int \{(\text{sen } x)^{-4}(1) - (\text{sen } x)^{-4}(\text{sen}^2 x)\} . \cos x \, dx =$$

$$\int \{(\text{sen } x)^{-4} - (\text{sen } x)^{-4+2}\} . \cos x \, dx =$$

$\int \{(\text{sen } x)^{-4} - (\text{sen } x)^{-2}\}.\cos x \, dx =$

$\int \{(\text{sen } x)^{-4}.\cos x - (\text{sen } x)^{-2}.\cos x\} \, dx =$

$\int (\text{sen } x)^{-4}.\cos x \, dx - \int (\text{sen } x)^{-2}.\cos x \, dx =$

Los diferenciales de ambas integrales estan completos.

$$\frac{(\text{sen } x)^{-4+1}}{-4+1} - \frac{(\text{sen } x)^{-2+1}}{-2+1} = \frac{(\text{sen } x)^{-3}}{-3} - \frac{(\text{sen } x)^{-1}}{-1} =$$

$$-\frac{1}{3(\text{sen } x)^3} + \frac{1}{(\text{sen } x)^1}$$. Por Trigonometría: $\frac{1}{\text{sen } x} = \csc x$

$- 1/3\ (\csc x)^3 + (\csc x) =$ **$\csc x - 1/3\ \csc^3 x + c$.**

7. $\int \frac{\text{sen}^3 \varphi}{\cos^2 \varphi} . d\varphi =$ **$\sec \varphi + \cos \varphi + c$.**

$\int (\cos \varphi)^{-2}.\text{sen}^2 \varphi \ . \text{sen } \varphi \ d\varphi = \int (\cos \varphi)^{-2}.(1 - \cos^2 \varphi).\text{sen } \varphi \ d\varphi$

$\int \{(\cos \varphi)^{-2}.(1 - \cos^2 \varphi)\}.\text{sen } \varphi \ d\varphi$

$\int \{(\cos \varphi)^{-2}.[1 - (\cos \varphi)^2]\}.\text{sen } \varphi \ d\varphi$

$\int \{(\cos \varphi)^{-2} - (\cos \varphi)^{-2}.(\cos \varphi)^2\}.\text{sen } \varphi \ d\varphi$

$\int \{(\cos \varphi)^{-2} - (\cos \varphi)^{-2+2}\}.\text{sen } \varphi \ d\varphi$

$\int \{(\cos \varphi)^{-2} - (\cos \varphi)^0\}.\text{sen } \varphi \ d\varphi = \int \{(\cos \varphi)^{-2} - 1\} . \text{sen } \varphi \, d\varphi =$

$\int \{(\cos \varphi)^{-2}.\text{sen } \varphi \; d\varphi - \int \text{sen } \varphi \; d\varphi =$

$\left.\begin{array}{l} v = \cos \varphi \\ dv = -\text{ sen } \varphi \; d\varphi \\ n = -2 \end{array}\right\}$ En la 1^{ra} integral, falta el signo (-) . La 2^{da} integral, su diferencial esta completo.

$(-)\int \{(\cos \varphi)^{-2}.(-)\text{ sen } \varphi \; d\varphi - \int \text{ sen } \varphi \; d\varphi =$

$-\dfrac{(\cos \varphi)^{-2+1}}{-2+1} - (-\cos \varphi) = -\dfrac{(\cos \varphi)^{-1}}{-1} + \cos \varphi = (\cos \varphi)^{-1} + \cos \varphi =$

$\dfrac{1}{(\cos \varphi)^{1}} + \cos \varphi = \mathbf{sec}\, \varphi + \mathbf{cos}\, \varphi + \mathbf{c}$ **.**

8. $\int \mathbf{cos^4 x . sen^3 x \, dx = -1/5 \, cos^5 x + 1/7 \, cos^7 x + c}$ **.**

$\int (\cos x)^4.\text{sen}^2 x .\text{sen } x \, dx = n\{(\cos x)^4.\text{sen}^2 x\}.\text{sen } x \, dx =$

$\int \{(\cos x)^4.(1 - \cos^2 x)\}.\text{sen } x \, dx =$

$\int \{(\cos x)^4.[1 - (\cos x)^2]\}.\text{sen } x \, dx =$

$\int \{(\cos x)^4 - (\cos x)^4.(\cos x)^2\}.\text{sen } x \, dx =$

$\int (\cos x)^4.\text{sen } x \, dx - \int (\cos x)^4(\cos x)^2.\text{sen } x \, dx =$

$\int (\cos x)^4.\text{sen } x \, dx - \int (\cos x)^6.\text{sen } x \, dx =$

En ambas integrales, les falta el signo (-) a sus diferenciales.

$(-) \{\int (\cos x)^4.(-)\text{sen } x \, dx\} - (-)\{\int (\cos x)^4(\cos x)^2.\text{sen } x \, dx\} =$

$-\dfrac{(\cos x)^{4+1}}{} + \dfrac{(\cos x)^{6+1}}{} = -\dfrac{(\cos x)^5}{} + \dfrac{(\cos x)^7}{}$

4+1 6+1 5 7

$- 1/5 (\cos x)^5 + 1/7 (\cos x)^7 = \mathbf{- 1/5 \cos^5 x + 1/7 \cos^7 x + c}$.

9. $\int \mathbf{sen^5 x\, dx = - \cos x + 2/3 \cos^3 x - 1/5 \cos^5 x + c}$ **.**

$\int sen^4 x .sen\, x\, dx = \int (sen^2 x)^2.sen\, x\, dx = \int (1 - \cos^2 x)^2.sen\, x\, dx =$

$\int (1 - 2\cos^2 x + \cos^4 x).sen\, x\, dx =$

$\int [1(sen\, x) - 2\cos^2 x .sen\, x + \cos^4 x .sen\, x]\, dx =$

$\int [sen\, x - 2(\cos x)^2.sen\, x + (\cos x)^4.sen\, x]\, dx =$

$\int sen\, x .\, dx - 2 \int (\cos x)^2.sen\, x .dx + \int (\cos x)^4.sen\, x\, dx =$
En la 1ª integral esta completo el diferencial.

Al 2do y 3er integral les falta el signo (-) a sus diferenciales.
$\int sen\, x .dx - 2(-)\{\int (\cos x)^2.(-)sen\, x.dx\} + (-) \{\int (\cos x)^4.(-)\, sen\, x$
$dx\}$

$\int sen\, x .dx - 2(-)\{\int (\cos x)^2.(-)\, sen\, x .dx\} + (-)\{\int (\cos x)^4.sen\, x\, dx\} =$

$$- \cos x + \frac{2(\cos x)^{2+1}}{2+1} - \frac{(\cos x)^{4+1}}{4+1} =$$

$$-\cos x + \frac{2(\cos x)^3}{} - \frac{(\cos x)^5}{} = - \cos x + 2/3 (\cos x)^3 - 1/5 (\cos x)^5$$

$\mathbf{- \cos x + 2/3 \cos^3 x - 1/5 \cos^5 x + c}$ **.**

10. $\int \mathbf{\cos^5 x\, dx = sen\, x - 2/3\, sen^3 x + 1/5\, sen^5 x + c}$ **.**

$\int \cos^4 x .\cos x\, dx = \int (\cos^2 x)^2.\cos x\, dx = \int (1 - sen^2 x)^2.\cos x\, dx =$

$\int (1 - 2sen^2 x + sen^4 x) .cos\ x\ dx =$

$\int [1(cos\ x) - 2sen^2 x.cos\ x + sen^4 x.cos\ x]\ dx =$

$\int (cos\ x - 2sen^2 x.cos\ x + sen^4 x.cos\ x)\ dx =$

$\int cos\ x\ dx - 2\int sen^2 x.cos\ x\ dx + \int sen^4 x.cos\ x\ dx =$

$\int cos\ x\ dx - 2\int (sen\ x)^2.cos\ x\ dx + \int (sen\ x)^4.cos\ x\ dx =$

La 1ra, 2da y 3ra integrales tienen sus diferenciales completos .

$\int cos\ x\ dx - 2\int (sen\ x)^2.cos\ x\ dx + \int (sen\ x)^4.cos\ x\ dx =$

$sen\ x - \frac{2(sen\ x)^{2+1}}{2+1} + \frac{(sen\ x)^{4+1}}{4+1} =$

$sen\ x - \frac{2(sen\ x)^3}{3} + \frac{(sen\ x)^5}{5} =$

$\mathbf{sen\ x - 2/3\ sen^3 x + 1/5\ sen^5 x + c}$

11. $\displaystyle \int \frac{\mathbf{sen^5\ y}}{\sqrt{\mathbf{cos\ y}}}\mathbf{dy}$.

$\int sen^4 y.sen\ y.(cos\ y)^{-1/2}\ dy = \int sen^4 y.(cos\ y)^{-1/2}.sen\ y\ dy =$

$\int (1 - cos^2 y)^2.(cos\ y)^{-1/2}.sen\ y\ dy =$

$\int \{[(1 - 2cos^2 y + cos^4 y).(cos\ y)^{-1/2}].sen\ y\}dy =$

$\int \{[(1 . (cos\ y)^{-1/2} - 2(cos\ y)^2.(cos\ y)^{-1/2} + (cos\ y)^4.(cos\ y)^{-1/2}].sen\ y\}dy =$

$\int \{[(cos\ y)^{-1/2} - 2(cos\ y)^{2-1/2} + (cos\ y)^{4-1/2}].sen\ y\}dy =$

$$\int \{(\cos y)^{-1/2}.\text{sen } y - 2(\cos y)^{3/2}.\text{sen } y + (\cos y)^{7/2}.\text{sen } y\}dy =$$

$$\int (\cos y)^{-1/2}.\text{sen } y\, dy - 2\int (\cos y)^{3/2}.\text{sen } y\, dy + \int (\cos y)^{7/2}.\text{sen } y\, dy$$

$$(-)\int (\cos y)^{-1/2}.(-)\,\text{sen } y\, dy - 2(-)\int (\cos y)^{3/2}.(-)\,\text{sen } y\, dy + (-)\int (\cos y)^{7/2}.(-)\,\text{sen } y\, dy$$

$$\frac{-(\cos y)^{-1/2+1}}{-1/2+1} + \frac{2(\cos y)^{3/2+1}}{3/2+1} - \frac{(\cos y)^{7/2+1}}{7/2+1}.$$

$$\frac{-(\cos y)^{1/2}}{1/2} + \frac{2(\cos y)^{5/2}}{5/2} - \frac{(\cos y)^{9/2}}{9/2}.$$

$$-2(\cos y)^{1/2} + \frac{4}{5}.(\cos y)^{5/2} - \frac{2}{9}.(\cos y)^{9/2}.$$

$$-2(\cos y)^{1/2} + 2.(\cos y)^{1/2}.\frac{2}{5}(\cos y)^{4/2} - 2(\cos y)^{1/2}.\frac{1}{9}.(\cos y)^{8/2} =$$

$$-2(\cos y)^{1/2}\{1 - \frac{2}{5}(\cos y)^{4/2} + \frac{1}{9}(\cos y)^{8/2}\} =$$

$$-2\sqrt{(\cos y)}\{1 - \frac{2}{5}(\cos y)^2 + \frac{1}{9}(\cos y)^4\} =$$

$$-2\sqrt{\cos y}\left(1 - \frac{2}{5}\cos^2 y + \frac{1}{9}\cos^4 y\right) + c.$$

12. $\displaystyle\int \frac{\cos^5 t}{\sqrt[3]{\text{sen} t}}\, dt = 3\,\text{sen}^{2/3} t\, (1 - 1/2\, \text{sen}^2 t + 1/7\, \text{sen}^4 t).$

$$\int \cos^4 t\, .\cos t(\text{sen } t)^{-1/3}.dt = \int (\cos^2 t)^2.\cos t(\text{sen } t)^{-1/3}.dt =$$

$$\int (1 - \text{sen}^2 t)^2.(\text{sen } t)^{-1/3}.\cos t\, dt =$$

$$\int \{(1 - 2\text{sen}^2 t + \text{sen}^4 t).(\text{sen } t)^{-1/3}\}.\cos t\, dt =$$

$\int \{(1)(\text{sen } t)^{-1/3} - 2\text{sen}^2 t.(\text{sen } t)^{-1/3} + \text{sen}^4 t.(\text{sen } t)^{-1/3}\}.\cos t\, dt =$

$\int \{(\text{sen } t)^{-1/3} - 2(\text{sen } t)^2.(\text{sen } t)^{-1/3} + (\text{sen } t)^4.(\text{sen } t)^{-1/3}\}.\cos t\, dt =$

$\int \{(\text{sen } t)^{-1/3} - 2(\text{sen } t)^{2-1/3} + (\text{sen } t)^{4-1/3}\}.\cos t\, dt =$

$\int \{(\text{sen } t)^{-1/3} - 2(\text{sen } t)^{5/3} + (\text{sen } t)^{11/3}\}.\cos t\, dt =$

$\int (\text{sen } t)^{-1/3}.\cos t\, dt - 2\int (\text{sen } t)^{5/3}.\cos t\, dt + \int (\text{sen } t)^{11/3}.\cos t\, dt =$

La 1ra , 2da y 3ra integrales tienen sus diferenciales completos .

$\dfrac{(\text{sen } t)^{-1/3+1}}{-1/3+1} - \dfrac{2(\text{sen } t)^{5/3+1}}{5/3+1} + \dfrac{(\text{sen } t)^{11/3+1}}{11/3+1} =$

$\dfrac{(\text{sen } t)^{2/3}}{2/3} - \dfrac{2(\text{sen } t)^{8/3}}{8/3} + \dfrac{(\text{sen } t)^{14/3}}{14/3} =$

$\dfrac{3}{2}.(\text{sen } t)^{2/3} - \dfrac{3}{}.\dfrac{\cancel{2}}{\cancel{8}}(\text{sen } t)^{8/3} + \dfrac{3}{14}.(\text{sen } t)^{14/3} =$

$\dfrac{3}{2}(\text{sen } t)^{2/3} - \dfrac{3}{4}(\text{sen } t)^{8/3} + \dfrac{3}{14}(\text{sen } t)^{14/3} =$

$\dfrac{3}{2}(\text{sen } t)^{2/3} - \dfrac{3}{2}(\text{sen } t)^{2/3}.\dfrac{1}{2}.(\text{sen } t)^{6/3} + \dfrac{3}{2}(\text{sen } t)^{2/3}.\dfrac{1}{7}.(\text{sen } t)^{12/3} =$

$\dfrac{3}{2}.(\text{sen } t)^{2/3}\left\{1 - \dfrac{1}{2}.(\text{sen } t)^{6/3} + \dfrac{1}{7}(\text{sen } t)^{12/3}\right\} =$

$\dfrac{3}{2}.(\text{sen } t)^{2/3}\left\{1 - \dfrac{1}{2}.(\text{sen } t)^{2} + \dfrac{1}{7}.(\text{sen } t)^{4}\right\} + c .$

$\dfrac{3}{2}.\text{sen}^{2/3} t\left\{1 - \dfrac{1}{2}.\text{sen}^2 t + \dfrac{1}{7}.\text{sen}^4 t\right\} + c .$

13. $\int \mathbf{sen^3\, 2\theta\ .\ d\theta}$.

$\int \text{sen}^2\, 2\theta\ .\ \text{sen}\ 2\theta\ d\theta = \int (1 - \cos^2 2\theta)\ .\ \text{sen}\ 2\theta\ d\theta =$

$\int [1(\text{sen}\ 2\theta) - \cos^2 2\theta.\text{sen}\ 2\theta]d\theta = \int (\text{sen}\ 2\theta - \cos^2 2\theta.\text{sen}\ 2\theta)d\theta$

$\int \text{sen}\ 2\theta\ d\theta - \int \cos^2 2\theta.\text{sen}\ 2\theta\ d\theta = \int \text{sen}\ 2\theta.d\theta - \int (\cos 2\theta)^2.\text{sen}$
$2\theta\, d\theta$

$\{v = 2\theta\ ;\ dv = 2\ d\theta\}\ \ ;\ \ \{v = \cos 2\theta\ ;\ dv = -\ 2\ \text{sen}\ 2\theta\ d\theta\}$

$\frac{1}{2}\ .\int \text{sen}\ 2\theta\ .\ (2)d\theta\ - (-\ \frac{1}{2}) \int (\cos 2\theta)^2\ .(-2)\ \text{sen}\ 2\theta\ d\theta =$

$\frac{1}{2}\ .\int \text{sen}\ 2\theta\ .\ (2)d\theta\ + \frac{1}{2} \int (\cos 2\theta)^2\ .(-2)\ \text{sen}\ 2\theta\ d\theta =$

$\frac{1}{2}\ .(-\cos 2\theta)\ + \frac{1}{2}\ .\ \frac{(\cos 2\theta)^{2+1}}{2+1} = -\frac{\cos 2\theta}{2} + \frac{1}{2}\ .\frac{(\cos 2\theta)^3}{3} =$

$-\frac{\cos 2\theta}{2} + \frac{(\cos 2\theta)^3}{6} + c$.

14. $\int \mathbf{\cos^3 \frac{\theta}{2}\ d\theta}$.

$\int \cos^2(\frac{1}{2}\theta).\cos(\frac{1}{2}\theta)\ d\theta = \int [1 - \text{sen}^2(\frac{1}{2}\theta)].\cos(\frac{1}{2}\theta)\ d\theta =$

$\int \cos(\frac{1}{2}\theta).d\theta - \int \text{sen}^2(\frac{1}{2}\theta).\cos(\frac{1}{2}\theta)\ d\theta =$

$2\int \cos(\frac{1}{2}\theta).\frac{1}{2}.d\theta - 2\int [\text{sen}(\frac{1}{2}\theta)]^2.\cos(\frac{1}{2}\theta)\ .\frac{1}{2}.d\theta =$

$2.\text{sen}(\frac{1}{2}\theta) - \frac{2\ [\text{sen}(\frac{1}{2}\theta)]^{2+1}}{2+1} = 2.\text{sen}(\frac{1}{2}\theta) - \frac{2\ [\text{sen}(\frac{1}{2}\theta)]^3}{3}$

$\mathbf{2\text{sen}(\frac{1}{2}\theta) - \frac{2\ [\text{sen}^3(\frac{1}{2}\theta)]}{3} + c}$

15. $\int$ **sen 2x cos 2x dx .**

$\int 2\text{sen } x \cos x .(\cos^2 x - \text{sen}^2 x)\, dx$.

$\int 2\text{sen } x \cos^3 x\, dx - \int 2\text{sen}^3 x \cos x\, dx = 2\int (\cos x)^3.\text{sen } x\, dx - 2\int (\text{sen } x)^3.\cos x\, dx$

Completando los diferenciales en ambas integrales:

$2(-)\int (\cos x)^3.(-)\text{sen}x - 2\int (\text{sen } x)^3.\cos x \;\; dx =$

$-2(\cos x)^{3+1} - 2(\text{sen } x)^{3+1} = \dfrac{-2(\cos x)^4}{4} - \dfrac{2(\text{sen } x)^4}{4} =$

$\dfrac{-(\cos x)^4}{2} - \dfrac{(\text{sen } x)^4}{2} = -\tfrac{1}{2}\{(\cos x)^4 + (\text{sen } x)^4\} + c$.

16. $\int$ **sen³ t cos³ t dt .**

$\int \text{sen}^2 t .\text{sen } t .\cos^2 t .\cos t\, dt = \int \text{sen}^2 t .\text{sen } t .(1 - \text{sen}^2 t) .\cos t\, dt$

$\int \text{sen}^3 t .\cos t.(1 - \text{sen}^2 t)\, dt = \int (\text{sen}^3 t .\cos t - \text{sen}^5 t .\cos t)\, dt$

$\int \text{sen}^3 t.\text{cos}t\, dt - \int \text{sen}^5 t.\text{cos}t\, dt = \int (\text{sen } t)^3.\text{cos}t\, dt - \int (\text{sen } t)^5.\text{cos}t\, dt$

Ambos diferenciales estan completos, se procede a integrar.

Se emplea: $\int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c$.

$\dfrac{(\text{sen } t)^{3+1}}{3+1} - \dfrac{(\text{sen } t)^{5+1}}{5+1} = \dfrac{(\text{sen } t)^4}{4} - \dfrac{(\text{sen } t)^6}{6} = \dfrac{\text{sen}^4 t}{4} - \dfrac{\text{sen}^6 t}{6} + c$.

Prueba por Diferenciación:

$$d\left\{\frac{(\text{sen}^4 t)}{4} - \frac{d}{dt}\frac{(\text{sen}^6 t)}{6} + \frac{d}{dt}(c)\right\}. dt$$

$$\left\{\frac{1}{\cancel{4}}.\cancel{4}.\ \text{sen}^3 t.\cos t - \frac{1}{\cancel{6}}.\cancel{6}.\text{sen}^5 t\ .\cos t\right\} dt$$

$$(\text{sen}^3 t.\cos t - \text{sen}^5 t\ .\cos t)\ .dt = [\cos t\ .\ \text{sen}^3 t\ (1 - \text{sen}^2 t)].dt =$$

$$[\cos t\ .\ \text{sen}^3 t\ .\cos^2 t].dt = [\text{sen}^3 t.\ \cos^3 t].dt\ .$$

Obteniendo asi el origen de la integral: $\int \text{sen}^3 t \cos^3 t\ dt$ **.**

17. $\int \cos^3 \frac{\phi}{2}\ \text{sen}^2 \frac{\phi}{2}\ d\phi$

$$\int (\cos^2 \tfrac{1}{2}\phi).\cos\tfrac{1}{2}\phi\ .\text{sen}^2 \tfrac{1}{2}\phi\ d\phi = \int \cos^2 \tfrac{1}{2}\phi.\text{sen}^2 \tfrac{1}{2}\phi.\cos\tfrac{1}{2}\phi\ d\phi =$$

$$\int (1 - \text{sen}^2 \tfrac{1}{2}\phi).\text{sen}^2 \tfrac{1}{2}\phi.\cos\tfrac{1}{2}\phi\ d\phi =$$
$$\int \text{sen}^2 \tfrac{1}{2}\phi.\cos \tfrac{1}{2}\phi - \text{sen}^2 \tfrac{1}{2}\phi.\text{sen}^2 \tfrac{1}{2}\phi.\cos \tfrac{1}{2}\phi\ d\phi$$

$\int \text{sen}^2 \tfrac{1}{2}\phi.\cos \tfrac{1}{2}\phi\ d\phi - \int \text{sen}^4 \tfrac{1}{2}\phi.\cos \tfrac{1}{2}\phi\ d\phi$.**Completando diferenciales.**

$$2\int (\text{sen}\ \tfrac{1}{2}\phi)^2.(\tfrac{1}{2})\cos \tfrac{1}{2}\phi\ .d\phi - 2\int (\text{sen}\ \tfrac{1}{2}\phi)^4.(\tfrac{1}{2})\cos \tfrac{1}{2}\phi\ d\phi.$$

$$\frac{2(\text{sen}\ \tfrac{1}{2}\phi)^{2+1}}{2+1} - \frac{2(\text{sen}\ \tfrac{1}{2}\phi)^{4+1}}{4+1} = \frac{2(\text{sen}\ \tfrac{1}{2}\phi)^3}{3} - \frac{2(\text{sen}\ \tfrac{1}{2}\phi)^5}{5} + c\ .$$

18. $\int \text{sen}^3 mt \cos^2 mt\ dt$ **.**

$$\int \text{sen}^3 mt \cos^2 mt\ dt = \int \text{sen}^2 mt\ \text{sen}\ mt \cos^2 mt\ dt =$$

$$\int (1 - \cos^2 mt).\ \cos^2 mt\ \text{sen}\ mt\ dt =$$

$\int (\cos^2 mt . \text{sen } mt - \cos^2 mt . \cos^2 mt . \text{sen } mt)\, dt =$

$\int \cos^2 mt . \text{sen } mt . dt - \int \cos^4 mt . \text{sen } mt\, dt .$

$\int (\cos mt)^2 . \text{sen } mt . dt - \int (\cos mt)^4 . \text{sen } mt . dt.$ Completando los diferenciales.

$(-1/m)\int (\cos mt)^2 .(-m)\text{sen } mt . dt -(-1/m)\int (\cos mt)^4 .(-m)\text{sen } mt . dt$

$$\frac{(-1/m)\, (\cos mt)^{2+1}}{2+1} + \frac{(1/m)\, (\cos mt)^{4+1}}{4+1} =$$

$$-\frac{(\cos mt)^3}{3m} + \frac{(\cos mt)^5}{5m} = -\frac{\cos^3 mt}{3m} + \frac{\cos^5 mt}{5m} + c .$$

18. $\int \text{sen}^5\, nx\, dx$

$\int \text{sen}^2 nx . \text{sen}^2 nx . \text{sen } nx\, dx = \int (1 - \cos^2 nx)(1 - \cos^2 nx) . \text{sen } nx\, dx$

$\int (1 - \cos^2 nx)^2 . \text{sen } nx\, dx = \int (1 - 2\cos^2 nx + \cos^4 nx) . \text{sen } nx\, dx$

$\int (\text{sen } nx - 2\cos^2 nx . \text{sen } nx + \cos^4 nx . \text{sen } nx)\, dx$

$\int \text{sen } nx\, dx - 2\int \cos^2 nx . \text{sen } nx\, dx + \int \cos^4 nx . \text{sen } nx\, dx .$

$(1/n)\int \text{sen } nx . (n)dx - 2(-1/n)\int (\cos nx) .(-n)\text{sen } nx\, dx +$
$(-1/n)\int (\cos nx)^4 (-n) . \text{sen } nx\, dx =$

$(1/n)\int \text{sen } nx .(n)dx + (2/n)\int (\cos nx).(-n)\text{sen } nx\, dx -$
$(1/n)\int (\cos nx)^4 (-n) . \text{sen } nx\, dx =$

$$(1/n)(- \cos nx) + \frac{(2/n)(\cos nx)^{2+1}}{2+1} - \frac{(1/n)\, (\cos nx)^{4+1}}{4+1}$$

$$\frac{(-\cos nx)}{n} + \frac{(2)(\cos nx)^3}{n(3)} - \frac{(1)(\cos nx)^5}{n(5)}$$

$$\frac{-\cos nx}{n} + \frac{2\cos^3 nx}{3n} - \frac{\cos^5 nx}{5n} + c\,.$$

20. $\int \cos^3 (a + bt)\, dt\,.$

$$\int \cos^2 (a + bt).\, \cos (a + bt)\, dt =$$

$$\int [1 - \operatorname{sen}^2 (a + bt)]\, .\cos (a + bt)\, dt$$

$$\int \cos (a + bt) - [\operatorname{sen}^2 (a + bt)].\, \cos (a + bt) dt\,.$$

$$\int \cos (a + bt)\, .\, dt - \int [\operatorname{sen} (a + bt)]^2.\, \cos (a + bt) dt\,.$$

Completando los diferenciales:

$$\left\{\begin{array}{l} v = (a + bt) \\ dv = b\, dt \end{array}\right\} \qquad \left\{\begin{array}{l} v = \operatorname{sen} (a + bt) \\ dv = [\cos (a + bt)].(b)\, dt \end{array}\right\}$$

$$(1/b)\int \cos (a + bt)\, .(b)\, dt - (1/b)\int [\operatorname{sen} (a + bt)]^2.(b) \cos (a + bt) dt$$

$$(1/b)\, .\operatorname{sen} (a + bt) - \frac{(1/b)\, [\operatorname{sen} (a + bt)]^{2+1}}{2+1} =$$

$$\frac{\operatorname{sen} (a + bt)}{b} - \frac{[\operatorname{sen} (a + bt)]^3}{b(3)} = \frac{\operatorname{sen} (a + bt)}{b} - \frac{[\operatorname{sen} (a + bt)]^3}{3b} + c\,.$$

$$\frac{\operatorname{sen} (a + bt)}{b} - \frac{[\operatorname{sen}^3 (a + bt)]}{3b} + c\,.$$

21. $\int \underline{\quad \cot \theta \quad}\, d\theta$

$\sqrt{\text{sen}\,\theta}$

Por Trigonometría: $\cot\theta = \dfrac{\cos\theta}{\text{sen}\,\theta}$.

$$\int \frac{\cos\theta}{\text{sen}\,\theta} \cdot \frac{1}{(\text{sen}\,\theta)^{1/2}} \cdot d\theta = \int \frac{\cos\theta}{(\text{sen}\,\theta)^{3/2}} \cdot d\theta = \int (\text{sen}\,\theta)^{-3/2} \cos\theta \cdot d\theta$$

El diferencial esta completo, se procede a integrar.

$$\frac{(\text{sen}\,\theta)^{-3/2+1}}{-3/2+1} = \frac{(\text{sen}\,\theta)^{-1/2}}{-1/2} = -2(\text{sen}\,\theta)^{-1/2} = \frac{-2}{(\text{sen}\,\theta)^{1/2}} =$$

$$= \frac{-2}{\sqrt{\text{sen}\,\theta}} + c\ .$$

22. $\displaystyle\int \frac{\text{sen}^3 2x}{\sqrt[3]{\cos 2x}}\, dx$

$$\int \frac{\text{sen}^3 2x}{(\cos 2x)^{1/3}}\, dx = \int \text{sen}^3 2x \cdot (\cos 2x)^{-1/3}\, dx =$$

$$\int \text{sen}^2 2x\, .\text{sen}\, 2x.(\cos 2x)^{-1/3}\, dx =$$

$$\int (1 - \cos^2 2x).\text{sen}\, 2x.(\cos 2x)^{-1/3} dx$$

$$\int (\cos 2x)^{-1/3}.\ \text{sen}\, 2x\ .\ (1 - \cos^2 2x)\, dx$$

$$\int \{(\cos 2x)^{-1/3}.\ \text{sen}\, 2x - [(\cos 2x)^{-1/3}.\ \text{sen}\, 2x.\ \cos^2 2x]\}\, dx$$

$$\int (\cos 2x)^{-1/3}.\ \text{sen}\, 2x - [(\cos 2x)^{-1/3}.\ \text{sen}\, 2x.\ (\cos 2x)^2]\}dx$$

$$\int (\cos 2x)^{-1/3}.\ \text{sen}\, 2x\, dx - \int [(\cos 2x)^{-1/3}\, .(\cos 2x)^{6/3}.\ \text{sen}\, 2x.]\}\, dx$$

$$\int (\cos 2x)^{-1/3}.\text{sen}\, 2x\, dx - \int [(\cos 2x)^{5/3}.\text{sen}\, 2x.]\}\, dx$$

$$\left\{\begin{array}{l} v = (\cos 2x) \\ dv = -2\text{sen } 2x\ .dx \\ n = -1/3 \end{array}\right\} \qquad \left\{\begin{array}{l} v = (\cos 2x) \\ dv = -2 \text{ sen } 2x\ dx \\ n = 5/3 \end{array}\right\}$$

$$(-1/2)\int (\cos 2x)^{-1/3}.(-2)\text{sen } 2x\ dx - (-1/2)\int [(\cos 2x)^{5/3}.(-2)\text{sen } 2x]\}dx$$

$$\frac{(-1/2).(\cos 2x)^{-1/3+1}}{-1/3+1} + \frac{(1/2).(\cos 2x)^{5/3+1}}{5/3+1} =$$

$$\frac{-(\cos 2x)^{2/3}}{2(2/3)} + \frac{(\cos 2x)^{8/3}}{2(8/3)} = \frac{-(\cos 2x)^{2/3}}{4/3} + \frac{(\cos 2x)^{8/3}}{16/3} =$$

$$\frac{-3(\cos 2x)^{2/3}}{4} + \frac{3(\cos 2x)^{8/3}}{16} = \frac{-3(\cos 2x)^{2/3}}{4} + \frac{3(\cos 2x)^{6/3}.(\cos 2x)^{2/3}}{16}$$

$$\frac{-3}{4}.(\cos 2x)^{2/3} + \frac{3}{4}.\frac{1}{4}.\underline{(\cos 2x)^{6/3}.(\cos 2x)^{2/3}} =$$

$$\frac{-3}{4}(\cos 2x)^{2/3}\left\{1 - \frac{1}{4}.(\cos 2x)^{6/3}\right\} = \frac{-3\sqrt[3]{(\cos 2x)^2}}{4}\left\{1 - \frac{(\cos 2x)^2}{2}\right\}$$

$$\frac{-3\sqrt[3]{(\cos 2x)^2}}{4}\left\{\frac{2 - (\cos 2x)^2}{2}\right\} = \frac{-3\sqrt[3]{\cos^2 2x}\ \{2 - (\cos 2x)^2\}}{8} + c\ .$$

Problemas. Páginas 262 y 263

Demostrar las siguientes Integraciones:

1. $\int tg^3 x\, dx = 1/2\, tg^2 x + \ln \cos x + c$.

$$\int tg^2 x . tg\, x\, dx = \int (sec^2 x - 1) . tg\, x\, dx = \int tg\, x . (sec^2 x - 1) . dx$$

$$\int (tg\, x .\ sec^2 x - tg\, x)\, dx = \int (tg\, x) . sec^2 x . dx - \int tg\, x\, dx$$

$$\left\{\begin{matrix} v = tg\, x \\ dv = sec^2 x\, dx \\ n = 1 \end{matrix}\right\} \qquad \left\{\begin{matrix} v = x \\ dv = dx \end{matrix}\right\}$$

$$\underline{(tg\, x)^{1+1}} - [- \ln (\cos x)] = \underline{(tg\, x)^2} + [\ \ln (\cos x)] =$$

1+1 2

$\frac{1}{2}(\text{tg } x)^2 + [\ \ln(\cos x)] + c$.

2. $\int \cot^3 \frac{x}{3}\, dx = -\frac{3}{2} \cdot \cot^2 \frac{x}{3} - 3 \ln \text{sen } x + c$.

Por Trigonometría: $\cot^2 x = \csc^2 x - 1$.

$$\int \cot^2 \frac{x}{3} \cdot \cot \frac{x}{3}\, dx = \int \cot \frac{x}{3} \cdot (\csc^2 \frac{x}{3} - 1)\, .dx =$$

$$\int \left\{\cot \frac{x}{3} \cdot \csc^2 \frac{x}{3} - \cot \frac{x}{3}\right\} . dx = \int \cot \frac{x}{3} \cdot \csc^2 \frac{x}{3} \cdot dx - \int \cot \frac{x}{3}\, .dx$$

$$\left\{\begin{array}{l} v = \cot \frac{x}{3} . \\ dv = -\frac{1}{3} . \csc^2 \frac{x}{3} \cdot dx \\ n = 1 \end{array}\right\} \qquad \left\{\begin{array}{l} v = \frac{x}{3} \\ dv = \frac{1}{3}\, dx \end{array}\right\}$$

$$(-3)\int \cot \frac{x}{3} \left(-\frac{1}{3}\right) \csc^2 \frac{x}{3} .dx - (3)\int \cot \frac{x}{3} \left(.\ \frac{1}{3}\right) dx =$$

$$-\frac{3\ (\cot {}_{1/3}x)^{1+1}}{1+1} - 3 \ln \text{sen}\left(\frac{x}{3}\right) = -\frac{3\ (\cot {}_{1/3}x)^2}{2} - 3 \ln \text{sen}\left(\frac{x}{3}\right) =$$

$$= -\frac{3}{2}\cot^2 \frac{x}{3} - 3 \ln \text{sen } \frac{x}{3} + c\ .$$

3. $\int \cot^3 2x \csc 2x\, dx = \frac{1}{2} \csc 2x - 1/6 \csc^3 2x + c$.

$$\int \cot 2x \cdot \cot^2 2x \cdot \csc 2x\, dx = \int \cot 2x \cdot \csc 2x . (\csc^2 2x - 1)\, dx =$$

$$\int (\cot 2x . \csc^2 2x \cdot \csc 2x - \cot 2x . \csc 2x)\, dx =$$

$\int \csc^2 2x \,.\, \csc 2x \,.\, \cot 2x \,.dx - \int \csc 2x \,.\, \cot 2x \,.\, dx =$

$$\left\{\begin{array}{l} v = (\csc 2x) \\ dv = [-\csc 2x \,.\, \cot 2x][2] \,.dx \\ dv = -2\csc 2x \,.\, \cot 2x \,.\, dx \\ n = 2 \end{array}\right\} \quad \left\{\begin{array}{l} v = 2x \\ dv = 2 \,.\, dx \end{array}\right\}$$

$(-\tfrac{1}{2})\left\{\int (\csc 2x)^2.(-2)\csc 2x.\cot 2x.dx\right\} - (\tfrac{1}{2})\left\{\int \csc 2x\,.\cot 2x.(2)\right\}$
dx

$$\frac{(-\tfrac{1}{2})(\csc 2x)^{2+1}}{2+1} - (\tfrac{1}{2})(-\csc 2x) = \frac{-(\csc 2x)^{2+1}}{2(3)} + \frac{(\csc 2x)}{2} =$$

$$-\frac{(\csc 2x)^3}{6} + \frac{(\csc 2x)}{2} = -\frac{(\csc^3 2x)}{6} + \frac{(\csc 2x)}{2} =$$

$$-\frac{1}{6}\csc^3 2x + \frac{1}{2}\csc 2x = \frac{1}{2}\csc 2x - \frac{1}{6}\csc^3 2x + c\,.$$

4. $\int \csc^4 \frac{x}{4}\, dx = -\frac{4}{3}\cot^3 \frac{x}{4} - 4\cot \frac{x}{4} + c\,.$

$$\int \csc^4 \frac{x}{4}\, dx = \int \csc^2 \frac{x}{4} \,.\, \csc^2 \frac{x}{4} dx = \int (\cot^2 \frac{x}{4} + 1)^2.\, \csc^2 \frac{x}{4} dx =$$

$$\int [\cot^2 \frac{x}{4}.\csc^2 \frac{x}{4} + \csc^2 \frac{x}{4}]dx = \int \cot^2 \frac{x}{4}.\csc^2 \frac{x}{4}\,.dx + \int \csc^2 \frac{x}{4}]dx$$

$$(-4)\int (\cot \frac{x}{4})^2.(-\frac{1}{4})\csc^2 \frac{x}{4}\,.dx + (4)\int \csc^2 \frac{x}{4}.\,(\frac{1}{4})dx$$

$$-\frac{4(\cot x/4\,)^{2+1}}{2+1} + 4(-\cot x/4) = -\frac{4(\cot x/4\,)^3}{3} - 4\cot x/4 =$$

$$-\frac{4}{3}(\cot x/4)^3 - 4\cot x/4 = -\frac{\mathbf{4}}{\mathbf{3}}\mathbf{\cot^3 \frac{x}{4} - 4 \cot \frac{x}{4}} + \mathbf{c}\,.$$

5. $\int \mathbf{tg^5 3\theta \; d\theta = 1/12\, tg^4 3\theta} - 1/6\, \mathbf{tg^2 3\theta + 1/3 \ln sec\, 3\theta + c}\,.$

$$\int tg^4 3\theta \,.\, tg\, 3\theta\, d\theta = \int tg^2 3\theta \,.\, (sec^2 3\theta - 1)\,.\, tg\, 3\theta\, d\theta =$$

$$\int tg^3 3\theta.(sec^2 3\theta - 1).d\theta = \int (tg^3 3\theta.sec^2 3\theta - tg^3 3\theta).d\theta =$$

$$\int tg^3 3\theta.sec^2 3\theta.d\theta - \int tg^3 3\theta.d\theta = \int tg^3 3\theta.sec^2 3\theta.d\theta - \int tg^2 3\theta.tg\, 3\theta\, d\theta$$

$$\int (tg\, 3\theta)^3\,.\, sec^2 3\theta\,.\, d\theta - \left\{ \int (sec^2 3\theta - 1).\, tg\, 3\theta\, d\theta \right\} =$$

$$\int (tg\, 3\theta)^3\,.\, sec^2 3\theta\,.\, d\theta - \left\{ \int (tg\, 3\theta\, .sec^2 3\theta - tg\, 3\theta)\, d\theta \right\} =$$

$$\int (tg\, 3\theta)^3\,.\, sec^2 3\theta\,.\, d\theta - \int (tg\, 3\theta)\,.\, sec^2 3\theta\, d\theta + \int tg\, 3\theta\, d\theta =$$

$$\left\{ \begin{array}{l} v = (tg\, 3\theta) \\ dv = 3sec^2 3\theta\, .d\theta \\ n = 3 \end{array} \right\} \quad \left\{ \begin{array}{l} v = (tg\, 3\theta) \\ dv = 3\, sec^2 3\theta\, .d\theta \\ n = 1 \end{array} \right\} \quad \left\{ \begin{array}{l} v = 3\theta \\ dv = 3\, .d\theta \end{array} \right\}$$

Se aplica en las dos primeras integrales $\left\{ \int v^n\, dv = \frac{v^{n+1}}{n+1} + c \right.$ **Se aplica en la 3^ra^ integral** $\left\{ tg\, v\, dv = \ln sec\, v + c\,. \right.$

$$(1/3)\int (tg\, 3\theta)^3.(3)sec^2 3\theta.d\theta - (1/3)\int (tg\, 3\theta).(3)sec^2 3\theta\, d\theta + (1/3)\int tg\, 3\theta\,.(3)d\theta$$

$$\frac{(1/3)(tg\, 3\theta)^{3+1}}{3+1} - \frac{(1/3)(tg\, 3\theta)^{1+1}}{1+1} + (1/3)\ln sec\, 3\theta$$

$$\frac{(tg\, 3\theta)^4}{3(4)} - \frac{(tg\, 3\theta)^2}{3(2)} + \frac{\ln sec\, 3\theta}{3} =$$

$$\underline{tg^4 3\theta} - \underline{tg^2 3\theta} + \underline{\ln sec\, 3\theta} = \underline{\;1\;}\mathbf{tg^4 3\theta} - \underline{\;1\;}\mathbf{tg^2 3\theta} + \underline{\;1\;}\mathbf{\ln sec\, 3\theta + c}$$

12 6 3 **12** **6** **3**

6. $\int \frac{\mathrm{sen}^2 \phi}{\cos^4 \phi}\, d\phi = \frac{1}{3}\, \mathrm{tg}^3 \phi + c$.

Por Trigonometría: $\mathrm{tg}^2 x = \sec^2 x - 1$; $\mathrm{sen}^2\phi / \cos^2\phi = \mathrm{tg}^2\phi$

$\int \frac{\mathrm{sen}^2 \phi}{\cos^2 \phi} \cdot \frac{1}{\cos^2 \phi} \cdot d\phi = \int \frac{\mathrm{sen}^2 \phi}{\cos^2 \phi} \cdot \sec^2 \phi \cdot d\phi =$

$\int \mathrm{tg}^2 \phi \cdot \sec^2 \phi \, .d\phi = \int (\mathrm{tg}\, \phi)^2 . \sec^2 \phi \, .d\phi =$

$\left\{ \begin{array}{l} v = (\mathrm{tg}\, \phi) \\ dv = \sec^2 \phi \\ n = 1 \end{array} \right\}$ El diferencial esta completo, se procede a integrar. Se aplica: $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$.

$\frac{(\mathrm{tg}\, \phi)^{2+1}}{2+1} = \frac{(\mathrm{tg}\, \phi)^3}{3} = \frac{(\mathrm{tg}^3 \phi)}{3} + c$.

7. $\int \frac{dx}{\mathrm{sen}^2 2x \cos^4 2x} = \mathrm{tg}\, 2x + 1/6\, \mathrm{tg}^3 2x - \frac{1}{2} \cot 2x + c$.

$\int \csc^2 2x \, . \sec^4 2x\, dx = \int \csc^2 2x \, . \sec^2 2x. \sec^2 2x\, dx =$

$\int \csc^2 2x. \sec^2 2x. (1 + \mathrm{tg}^2 2x) dx$

$\int (\csc^2 2x. \sec^2 2x + \csc^2 2x. \sec^2 2x. \mathrm{tg}^2 2x) dx =$

$\int [\csc^2 2x. (1 + \mathrm{tg}^2 2x) + \csc^2 2x. \mathrm{tg}^2 2x. (1 + \mathrm{tg}^2 2x)] dx =$

$\int (\csc^2 2x + \csc^2 2x\, . \mathrm{tg}^2 2x + \csc^2 2x\, . \mathrm{tg}^2 2x + \csc^2 2x\, . \mathrm{tg}^2 2x\, \mathrm{tg}^2 2x)\, dx$

$\int (\csc^2 2x + 2 \csc^2 2x\, . \mathrm{tg}^2 2x + \csc^2 2x\, . \mathrm{tg}^2 2x\, .\, \mathrm{tg}^2 2x)\, dx$

$\int (\csc^2 2x + 2 \csc^2 2x\, . \mathrm{tg}^2 2x + \csc^2 2x\, . \mathrm{tg}^2 2x\, .\, (\sec^2 2x - 1))\, dx$

$\int (\csc^2 2x + 2\csc^2 2x\,.tg^2 2x + \csc^2 2x\,.tg^2 2x.\sec^2 2x - \csc^2 2x\,.tg^2 2x)\,dx$

$\int (\csc^2 2x + \cancel{2\csc^2 2x\,.tg^2 2x} + \csc^2 2x\,.tg^2 2x.\sec^2 2x - \cancel{\csc^2 2x\,.tg^2 2x})\,dx$

$\int (\csc^2 2x + \csc^2 2x\,.tg^2 2x + \csc^2 2x\,.tg^2 2x.\sec^2 2x\,)\,dx$

$\int [\csc^2 2x + (1 + \cot^2 2x).tg^2 2x + tg^2 2x.\,\sec^2 2x\,.(1 + \cot^2 2x)]\,dx$

$\int [\csc^2 2x + tg^2 2x + \cot^2 2x.\,tg^2 2x + tg^2 2x.\sec^2 2x + tg^2 2x.\,\sec^2 2x\,.\cot^2 2x)]$

dx

Por Trigonometría: $\begin{cases} tg\,x\,.\,\cot x = 1 \;\square\; tg^2 x\,.\,\cot^2 x = 1 \\ 1 + tg^2 x = \sec^2 x \end{cases}$

$\int [\csc^2 2x + tg^2 2x + (1) + tg^2 2x.\,\sec^2 2x + \sec^2 2x\,.(1)\,]\,dx$

$\int [\csc^2 2x + tg^2 2x + 1 + tg^2 2x.\,\sec^2 2x + \sec^2 2x\,]\,dx$

$\int [\csc^2 2x + (tg^2 2x + 1) + tg^2 2x.\,\sec^2 2x + \sec^2 2x\,]\,dx$

$\int [\csc^2 2x + \sec^2 2x + tg^2 2x.\,\sec^2 2x + \sec^2 2x\,]\,dx$

$\int [\csc^2 2x + \cancel{\sec^2 2x} + tg^2 2x.\,\sec^2 2x + \cancel{\sec^2 2x}\,]\,dx$

$\int [\csc^2 2x + 2\sec^2 2x + tg^2 2x.\,\sec^2 2x]\,dx$

$\int \csc^2 2x\,.\,dx + 2\int \sec^2 2x\,.\,dx + \int tg^2 2x.\,\sec^2 2x\,.\,dx$

$\int \csc^2 2x\,.\,dx + 2\int \sec^2 2x\,.\,dx + \int (tg\,2x)^2.\,\sec^2 2x\,.\,dx$

$\begin{Bmatrix} v = 2x \\ dv = 2\,dx \end{Bmatrix} \qquad \begin{Bmatrix} v = 2x \\ dv = 2\,dx \end{Bmatrix} \qquad \begin{Bmatrix} v = tg\,2x \\ dv = 2\,\sec^2 2x\,.\,dx \\ n = 2 \end{Bmatrix}$

Para la 1ra integral, se aplica: $\int \csc^2 v\,.dv = -\cot v + c$.
Para la 2da integral, se aplica: $\int \sec^2 v\,dv = tg\,v + c$.
Para la 3ra integral, se aplica: $\int v^n.\mathbf{dv} = \dfrac{v^{n+1}}{n+1}\,dx$

$(1/2)\int \csc^2 2x\,.(2)\,dx + 2(1/2)\int \sec^2 2x\,.(2)\,dx + (1/2)\int (tg\ 2x)^2.(2)\sec^2 2x\ .$
dx

$$(1/2)(-\cot 2x) + 2(1/2)(tg\ 2x) + \frac{1}{2}.\frac{(tg\ 2x)^{2+1}}{2+1} =$$

$$-1/2\cot 2x + tg\ 2x + \frac{(tg\ 2x)^3}{2(3)} = -1/2\cot 2x + tg\ 2x + \frac{tg^3 2x}{6}$$

Ordenando: $tg\ 2x + \frac{1}{6}tg^3\ 2x - \frac{1}{2}\cot 2x + c$.

8. $$\int \frac{\cos^4 x}{sen^6 x}dx = -1/5\ ctg^5 x + c\ .$$

$$\int \frac{\cos^4 x}{sen^4 x}.\frac{1}{sen^2 x}dx = \int \cot^4 x\,.\csc^2 x\ .\ dx = \int (\cot x)^4\ .\ \csc^2 x\ .$$
dx

$$\left\{\begin{array}{l} v = \cot x \\ dv = -\csc^2 x\,dx \end{array}\right\} \text{Se aplica: } \int v^n\,.dv = \frac{v^{n+1}}{n+1} + c\ .$$

$$(-)\int (\cot x)^4.(-)\csc^2 x\,.dx = \frac{(-)(\cot x)^{4+1}}{4+1} = \frac{-\cot^5 x)}{5} = -\frac{1}{5}\cot^5 x + c$$

9. $$\int \frac{sen^{3/2} x}{\cos^{11/2} x}dx = 2/5\ tg^{5/2} x + 2/9\ tg^{9/2} x + c\ .$$

$$\int \frac{sen^{3/2} x}{\cos^{11/2} x}dx = \int \frac{sen^{3/2} x}{\cos^{3/2} x}.\frac{1}{\cos^{8/2} x}.dx = \int tg^{3/2} x\ .\frac{1}{\cos^4 x}.dx$$

$$\int tg^{3/2} x.\sec^4 x.dx = \int tg^{3/2} x.\sec^2 x.\sec^2 x.dx =$$

$$\int tg^{3/2} x.\sec^2 x.(1 + tg^2 x).dx =$$

$$\int (tg^{3/2} x.\sec^2 x + tg^{3/2} x\ .\ \sec^2 x.\ tg^2 x)\ .\ dx =$$

$\int (tg^{3/2}\, x\, .\, sec^2\, x + tg^{3/2}\, x.sec^2\, x.tg^{4/2}\, x)\, .\, dx =$

$\int (tg^{3/2}\, x.sec^2\, x + tg^{7/2}\, x.sec^2\, x).dx =$

$\int tg^{3/2}\, x.sec^2\, x.dx + \int tg^{7/2}\, x.sec^2\, x.dx =$

El diferencial esta completo en ambas integrales.
Se aplica: $\int v^n\, .dv = \dfrac{v^{n+1}}{n+1} + c$.

$\int (tg\, x)^{3/2}\, .\, sec^2\, x.\, dx + \int (tg\, x)^{7/2}\, .\, sec^2\, x\, .\, dx =$

$\dfrac{(tg\, x)^{3/2+1}}{3/2+1} + \dfrac{(tg\, x)^{7/2+1}}{7/2+1} = \dfrac{(tg\, x)^{5/2}}{5/2} + \dfrac{(tg\, x)^{9/2}}{9/2} =$

$\dfrac{2}{5}(tg\, x)^{5/2} + \dfrac{2}{9}(tg\, x)^{9/2} = \mathbf{\dfrac{2}{5}\, tg^{5/2}\, x + \dfrac{2}{9}\, tg^{9/2}\, x + c}$.

10. $\int \mathbf{tg^3\, \alpha + sec^{5/2}\, \alpha\, .\, d\alpha = \dfrac{2}{9}\, sec^{5/2}\, \alpha - \dfrac{2}{5}\, sec^{5/2}\, \alpha + c}$.

$\int tg^3\, \alpha\, .\, sec^{5/2}\, \alpha\, .\, d\alpha = \int (tg^2\, \alpha.\, tg\, \alpha\, .\, sec^{5/2}\, \alpha)\, .\, d\alpha =$
$\int [(sec^2\, \alpha - 1).\, tg\, \alpha\, .\, sec^{5/2}\, \alpha]\, .\, d\alpha =$

$\int [sec^2\, \alpha\, .\, sec^{5/2}\, \alpha\, .tg\, \alpha - sec^{5/2}\, \alpha\, .\, tg\, \alpha]\, .\, d\alpha =$

$\int [sec^{9/2}\, \alpha\, .\, tg\, \alpha - sec^{3/2}\, \alpha\, .\, sec^{2/2}\, \alpha\, .\, tg\, \alpha]\, .\, d\alpha =$

$\int [sec^{7/2}\, \alpha\, .\, sec^{2/2}\, \alpha\, .\, tg\, \alpha - sec^{3/2}\, \alpha\, .\, sec\, \alpha\, .\, tg\, \alpha]\, .\, d\alpha =$
$\int sec^{7/2}\, \alpha\, .\, sec\, \alpha\, .\, tg\, \alpha\, .\, d\alpha - \int sec^{3/2}\, \alpha\, .\, sec\, \alpha\, .\, tg\, \alpha\, .\, d\alpha =$
$\int (sec\, \alpha)^{7/2}\, .\, sec\, \alpha\, .tg\, \alpha\, .\, d\alpha - \int (sec\, \alpha)^{3/2}\, .\, sec\, \alpha\, .\, tg\, \alpha\, .\, d\alpha =$

$\left\{ \begin{array}{l} v = sec\, \alpha \\ dv = sec\, \alpha\, .tg\, \alpha\, .\, d\alpha \end{array} \right\}$ $\left\{ \begin{array}{l} v = sec\, \alpha \\ dv = sec\, \alpha\, .tg\, \alpha\, .\, d\alpha \end{array} \right\}$

Ambos diferenciales estan completos, se aplica en ambos: $\int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c$

$$\frac{(\sec\alpha)^{7/2+1}}{7/2+1} - \frac{(\sec\alpha)^{3/2+1}}{3/2+1} = \frac{(\sec\alpha)^{9/2}}{9/2} - \frac{(\sec\alpha)^{5/2}}{5/2} =$$

$$\frac{2(\sec\alpha)^{9/2}}{9} - \frac{2(\sec\alpha)^{5/2}}{5} = \frac{2}{9}(\sec\alpha)^{9/2} - \frac{2}{5}(\sec\alpha)^{5/2} =$$

$$\frac{2}{9}\sec^{9/2}\alpha - \frac{2}{5}\sec^{5/2}\alpha + c\,.$$

11. $$\int \left(\frac{\sec ax}{\operatorname{tg} ax}\right)^4 . dx = -\frac{1}{a}\left(\cot ax\right) + \frac{1}{3}\cot^3 ax + c\,.$$

Por Trigonometría: $\sec v = \dfrac{1}{\cos v}$; $\cot v = \dfrac{1}{\operatorname{tg} v}$; $\csc v = \dfrac{1}{\operatorname{sen} v}$; $\csc^2 v = 1 + \cot^2 v$.

$$\int \frac{\sec^4 ax}{\operatorname{tg}^4 ax} . dx = \int \frac{1}{\cos^4 ax} . \frac{1}{\operatorname{tg}^4 ax} . dx = \int \frac{1}{\cos^4 ax} . \cot^4 ax . dx =$$

$$\int \frac{1}{\cos^4 ax} . \frac{\cos^4 ax}{\operatorname{sen}^4 ax} . dx = \int \frac{1}{\cancel{\cos^4 ax}} . \frac{\cancel{\cos^4 ax}}{\operatorname{sen}^4 ax} . dx = \int \frac{1}{\operatorname{sen}^4 ax} dx =$$

$$\int \csc^4 ax\, dx = \int \csc^2 ax . \csc^2 ax\, dx = \int \csc^2 ax . (1 + \cot^2 ax)\, dx =$$

$$\int (\csc^2 ax + \cot^2 ax . \csc^2 ax)\, dx = \int \csc^2 ax\, dx + \int \cot^2 ax . \csc^2 ax\, dx$$

=

$$(1/a) . \int \csc^2 ax . (a)dx + (-1/a) . \int (\cot ax)^2 . (-a)\csc^2 ax\, dx =$$

$$\left\{ \begin{array}{l} v = ax \\ dv = a\, dx \end{array} \right\} \qquad \left\{ \begin{array}{l} v = (\cot ax) \\ dv = a.\csc^2 ax\, dx \end{array} \right\}$$

Para la 1ra integral, aplicamos: $\int \csc^2 v . dv = -\cot v + c$.

Para la 2^{da} integral, aplicamos: $\int v^n .dv = \frac{v^{n+1}}{n+1} + c.$

$$(1/a)\,(-\cot ax) - \frac{(1/a)(\cot ax)^{2+1}}{2+1} = -\frac{1}{a}.\cot ax - \frac{1}{a}.\frac{(\cot ax)^{2+1}}{2+1} =$$

$$= -\frac{1}{a}.\cot ax - \frac{1}{a}.\frac{(\cot ax)^3}{3} = -\frac{1}{a}.\cot ax - \frac{1}{a}.\frac{\cot^3 ax}{3}$$

$$= -\frac{1}{a}\left(\cot x + \frac{1}{3}\cot^3 ax\right) + c\,.$$

12. $\int (\cot^2 2\theta + \cot^4 2\theta)\,.\,d\theta = -1/6\cot^3 2\theta + c\,.$

$\int (\cot^2 2\theta + \cot^2 2\theta\,.\,\cot^2 2\theta)\,d\theta$. Por Trigonometría: $\cot^2 v = \csc^2 v - 1$.

$$\int [\cot^2 2\theta + \cot^2 2\theta\,.(\csc^2 2\theta - 1)]\,d\theta$$

$$\int (\cot^2 2\theta + \cot^2 2\theta\,.\,\csc^2 2\theta - \cot^2 2\theta)\,d\theta$$

$$\int (\cancel{\cot^2 2\theta} + \cot^2 2\theta\,.\,\csc^2 2\theta - \cancel{\cot^2 2\theta})\,d\theta$$

$$\int \cot^2 2\theta\,.\,\csc^2 2\theta\,.\,d\theta = (-½)\,.\int (\cot 2\theta)^2\,.(-2)\csc^2 2\theta\,.\,d\theta$$

$$\frac{(-½)(\cot 2\theta)^{2+1}}{2+1} = -\frac{1}{2}\,.\,\frac{(\cot 2\theta)^3}{3} = -\frac{1}{6}\cot^3 2\theta + c\,.$$

13. $\int (tg\, bt - \cot bt)^3\,dt = \frac{1}{2b}[tg^2 bt + \cot^2 bt] + \frac{4}{b}\ln sen\,2\,bt + c\,.$

$$\int (tg^3 bt - 3tg^2 bt\,.\cot bt + 3\,tg\,bt\,.\cot^2 bt - \cot^3 bt)dt$$

$$\int (tg^3 bt - 3tg\,bt\,.\cancel{tg\,bt}\,.\frac{1}{\cancel{tg\,bt}} + 3.\frac{1}{\cancel{\cot bt}}.\cancel{\cot bt}\,.\cot bt - \cot^3 bt)dt$$

$\int (tg^3\, bt - 3tg\, bt + 3cot\, bt - cot^3\, bt)dt$

$\int (tg^2\, bt \,.\, tg\, bt - 3tg\, bt + 3cot\, bt - cot^2\, bt \,.\, cot\, bt)dt$

$\int [(sec^2\, bt - 1) \,.\, tg\, bt - 3tg\, bt + 3cot\, bt - (csc^2\, bt - 1) \,.\, cot\, bt]dt$

$\int [sec^2\, bt \,.\, tg\, bt - tg\, bt - 3tg\, bt + 3cot\, bt - csc^2\, bt \,.\, cot\, bt + cot\, bt]dt$

$\int [sec^2\, bt \,.\, tg\, bt - 4tg\, bt + 4cot\, bt - csc^2\, bt \,.\, cot\, bt\,]dt$

$\int tg\, bt \,.\, sec^2\, bt \,.dt - 4\int tg\, bt \,.\, dt + 4\int cot\, bt \,.\, dt - \int cot\, bt \,.\, csc^2\, bt \,.$
dt

$(1/b) \int (tg\, bt)^1.(b)sec^2\, bt \,.\, dt - 4(1/b)\int tg\, bt\, .(b)\, dt +$

$4(1/b)\int cot\, bt\, .(b)\, dt - (-1/b)\int (cot\, bt)^1 \,.\, (-b)csc^2\, bt \,.\, dt$

$$\frac{1}{b}\frac{(tg\, bt)^{1+1}}{1+1} - \frac{4}{b}[-\ln\cos bt] + \frac{4}{b}[\ln sen\, bt] + \frac{1}{b}\frac{(cot\, bt)^{1+1}}{1+1} =$$

$$\frac{1}{b}\,.\,\frac{(tg\, bt)^2}{2} + \frac{4}{b}\,.\ln\cos bt + \frac{4}{b}[\ln sen\, bt] + \frac{1}{b}\,.\,\frac{(cot\, bt)^2}{2}$$

$$\frac{1}{2b}\,.\,tg^2\, bt + \frac{4}{b}\,.\ln\cos bt + \frac{4}{b}\,.\ln sen\, bt + \frac{1}{2b}\,.\,cot^2\, bt.$$

<u>**Ordenando**</u>:

$$\frac{1}{2b}\,.\,tg^2\, bt + \frac{1}{2b}\,.\,cot^2\, bt + \frac{4}{b}\,.\ln\cos bt + \frac{4}{b}\,.\ln sen\, bt$$

$$\frac{1}{2b}\left(tg^2\, bt + cot^2\, bt\right) + \frac{4}{b}\left(\ln\cos bt + \ln sen\, bt\right)$$

$$\frac{1}{2b}\ tg^2\, bt + cot^2\, bt\ + \frac{4}{b}\ \ln \cos bt\ .sen\ bt$$

14. $\int$ **cot^5 ax dx .**

$$\int cot^4 ax.\ cot\ ax\ dx = \int cot^2 ax.\ cot^2 ax\ .\ cot\ ax\ dx =$$

$$\int (csc^2 ax - 1).\ (csc^2 ax - 1).\ cot\ ax\ dx = \int (csc^2 ax - 1)^2 .cot\ ax\ dx =$$

$$\int [csc^4 ax - 2csc^2 ax + 1].\ cot\ ax\ dx =$$

$$\int [csc^3 ax\ .\ csc\ ax\ .cot\ ax - 2csc\ ax\ .csc\ ax\ .cot\ ax + cot\ ax].dx =$$

$$(-1/a)\int (csc\ ax)^3.[-csc\ ax\ .cot\ ax\ .(a)]dx - 2(-1/a)\int (csc\ ax).[-csc\ ax.\ cot\ ax.(a)]dx + (1/a)\int cot\ ax\ .(a)dx\ .$$

$$-\frac{(-1/a)\ (csc\ ax)^{3+1}}{4} + \frac{2(1/a)\ (csc\ ax)^{1+1}}{2} + (1/a)\ln sen\ ax$$

$$\frac{(csc\ ax)^{3+1}}{4a} + \frac{\cancel{2}\ (csc\ ax)^{1+1}}{\cancel{2}\ a} + \frac{\ln sen\ ax}{a} + c\ .$$

15. $\int$ **$sec^6\ \theta$ dθ**

$$\int sec^4\ \theta\ .\ sec^2\ \theta\ d\theta = \int (tg^2\ \theta + 1)^2\ .\ sec^2\ \theta\ d\theta =$$
$$\int (tg^4\ \theta + 2\ tg^2\ \theta + 1)^2\ .\ sec^2\ \theta\ d\theta =$$

$$\int tg^4\ \theta.sec^2\ \theta\ d\theta + 2\int tg^2\ \theta.sec^2\ \theta\ d\theta + \int sec^2\ \theta\ d\theta =$$

$$\int (tg\ \theta)^4.sec^2\ \theta\ d\theta + 2\int (tg\ \theta)^2.sec^2\ \theta\ d\theta + \int sec^2\ \theta\ d\theta =$$

$$\frac{(tg\ \theta)^{4+1}}{4+1} + 2\ \frac{(tg\ \theta)^{2+1}}{2+1} + tg\ \theta = \frac{tg^5\ \theta}{5} + \frac{2\ tg^3\ \theta}{3} + tg\ \theta + c\ .$$

16. $\int$ **csc^6 x dx**

2

$\int (\csc^4 \frac{x}{2}\, dx.\ \csc^2 \frac{x}{2})\, dx = \int (1+\cot^2 \frac{x}{2})^2.\ \csc^2 \frac{x}{2}\ .dx$

$\int (1 + 2\cot^2 \frac{x}{2} + \cot^4 \frac{x}{2})\ .\ \csc^2 \frac{x}{2}\ .dx$

$\int \csc^2 \frac{x}{2}\ .dx + 2\int \cot^2 \frac{x}{2}\ .\ \csc^2 \frac{x}{2}\ .dx + \int \cot^4 \frac{x}{2}\ .\ \csc^2 \frac{x}{2}\ .dx$

$(2)\int \csc^2 \frac{1}{2} x\ .(\frac{1}{2})dx + 2(2)\int \cot^2 \frac{1}{2} x\ .\ \csc^2 \frac{1}{2} x\ .(\frac{1}{2})dx +$
$(2)\int \cot^4 \frac{1}{2} x\ .\csc^2 \frac{1}{2} x\ .\frac{1}{2}dx$

$\left.\begin{array}{l} v = \frac{1}{2} x \\ dv = \frac{1}{2} dx \end{array}\right\}$ **Falta ½ para completar el diferencial en la 1ra integral.**

$\left.\begin{array}{l} v = (\cot \frac{1}{2} x) \\ dv = -\frac{1}{2} \csc^2 \frac{1}{2} x\ .dx \\ n = 1 \end{array}\right\}$ **Falta (- ½) para completar el diferencial, en la 2da integral .**

$(2)\int \csc^2 \frac{1}{2} x\ .(\frac{1}{2})dx + 2(-2)\int (\cot \frac{1}{2} x)^2\ .\ \csc^2 \frac{1}{2} x\ .(-\frac{1}{2})dx +$
$(-2)\int (\cot \frac{1}{2} x)^4\ .\csc^2 \frac{1}{2} x\ .(-\frac{1}{2})dx$

$= -2\cot \frac{1}{2} x - \frac{4\ (\cot \frac{1}{2} x)^{2+1}}{2+1} - \frac{2\ (\cot \frac{1}{2} x)^{4+1}}{4+1} + c$

$= -2\cot \frac{1}{2} x - \frac{4(\cot \frac{1}{2} x)^3}{3} - \frac{2(\cot \frac{1}{2} x)^5}{5} + c$

17. $\int \frac{\sec^4 t}{tg^3\, t}\, dt$

$\int \sec^4 t.\cot^3 t\ dt = \int \sec^4 t.\cot^3 t\ dt = \int \sec^2 t.\ \sec^2 t.\cot^3 t\ dt =$

$\int (1 + tg^2 t)\ (1 + tg^2 t).\cot^3 t\ dt = \int (1 + tg^2 t)^2.\cot^3 t\ dt =$

$\int (1 + 2tg^2 t + tg^4 t).cot^3 t\, dt =$

$\int cot^3 t\, dt + 2\int tg^2 t.cot^3 t\, dt + \int tg^4 t.cot^3 t\, dt =$

$\int cot^3 t\, dt + 2\int tg^2 t.cot^2 t\, .\, cot\, t\, dt + \int tg^3 t.\, tg\, t.cot^3 t\, dt =$

$\int cot^3 t\, dt + 2\int tg^2 t\, .\frac{1}{tg^2 t}.cot\, t\, dt + \int tg^3 t.\frac{1}{tg^3 t}.tg\, t\, dt =$

$\int cot^3 t\, dt + \cancel{2\int tg^2} t\, .\frac{1}{\cancel{tg^2 t}}.cot\, t\, dt \cancel{+ \int tg^3} t.\frac{1}{\cancel{tg^3 t}}.tg\, t\, dt =$

$\int cot^3 t\, dt + 2\int cot\, t\, dt + \int tg\, t\, dt = \int cot^2 t\, .cot\, t\, dt + 2\int cot\, t\, dt + \int tg\, t\, dt$

Por Trigonometría: $cot^2 t = csc^2 t - 1$,reemplazando en la integral.

$\int (csc^2 t - 1).cot\, t\, dt + 2\int cot\, t\, dt + \int tg\, t\, dt =$

$\int csc^2 t\, .cot\, t\, dt - \int cot\, t\, dt + 2\int cot\, t\, dt + \int tg\, t\, dt$.
Simplificando:

$\int (cot\, t)\, .csc^2 t\, dt + \int cot\, t\, dt + \int tg\, t\, dt =$

$v = (cot\, t)$
$dv = - csc^2 t\, dt$
Falta (-) para completar el diferencial, en la 1ra integral.Se aplica: $\int v^n\, dv = v^{n+1} / n+1 + c$
La 2da y 3ra integral, estan listas para ser integradas.
$(-)\int (cot\, t)\, .(-)csc^2 t\, dt + \int cot\, t\, dt + \int tg\, t\, dt =$

$- \frac{(cot\, t)^{1+1}}{1+1} + \ln sen\, t + \ln sec\, t = - \frac{(cot\, t)^2}{2} + \ln sen\, t + \ln sec\, t + c$

Otra solución:

$\int \csc^2 t\,.\cot t\,dt + \int \cot t\,dt + \int tg\,t\,dt =$

$\int (\csc t)\,.\,\csc t\,.\cot t\,dt + \int \cot t\,dt + \int tg\,t\,dt =$

$\left.\begin{array}{l} v = \csc t \\ dv = -\csc t.\cot t\,dt \\ n = 1 \end{array}\right\}$ Falta (-) para completar el diferencial. Se aplica: $\int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c$.

$(-)\int \csc t\,.\,(-)\csc t\,.\cot t\,dt + \int \cot t\,dt + \int tg\,t\,dt =$

$-\dfrac{(\csc t)^{1+1}}{1+1} + \ln sen\,t + \ln sec\,t = \mathbf{-\dfrac{(\csc t)^2}{2} + \ln sen\,t + \ln sec\,t + c}$.

18. $\displaystyle \int \frac{\sec^4 x\;dx}{\sqrt{tg\,x}}$

$\int \sec^2 x.\,\sec^2 x.(tg\,x)^{-1/2}\,dx =$

$\int (1 + tg^2 x).\,\sec^2 x.(tg\,x)^{-1/2}\,dx$

$\int \sec^2 x.(tg\,x)^{-1/2}\,dx \;+ \int tg^2 x.(tg\,x)^{-1/2}.\,\sec^2 x.\,dx$

$\int (tg\,x)^{-1/2}.\,\sec^2 x\,dx \;+ \int (tg\,x)^{3/2}.\,\sec^2 x.\,dx$

$\left.\begin{array}{l} v = (tg\,x) \\ dv = \sec^2 x\,dx \\ n = -1/2 \end{array}\right\}$ 1ra integral. La integral esta completa. Se aplica: $\int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c$

$\left.\begin{array}{l} v = (tg\,x) \\ dv = \sec^2 x\,dx \\ n = 3/2 \end{array}\right\}$ 2da integral. La integral esta completa. Se aplica: $\int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c$

$\dfrac{(tg\,x)^{-1/2+1}}{-1/2+1} + \dfrac{(tg\,x)^{3/2+1}}{3/2+1} = \dfrac{(tg\,x)^{1/2}}{1/2} + \dfrac{(tg\,x)^{5/2}}{5/2} = \mathbf{2(tg\,x)^{1/2} + \dfrac{2(tg\,x)^{5/2}}{5} + c}$

19. $\int \left(\dfrac{\csc ax}{\cot ax}\right)^4 dx$

$\int \csc^4 ax.tg^4\,ax dx = \int \dfrac{1}{\cancel{sen^4 ax}} \cdot \dfrac{\cancel{sen^4 ax}}{\cos^4 ax} dx = \int \dfrac{1}{\cos^4 ax} dx = \int \sec^4 ax$
dx

$\int \sec^2 ax.\ \sec^2 ax\ dx = \int (1 + tg^2\ ax).\sec^2 ax\ dx =$

$\int \sec^2 ax\ dx + \int tg^2\ ax.\sec^2 ax\ dx =$

$\int \sec^2 ax\ dx + \int (tg\ ax)^2.\sec^2 ax\ dx =$

$(1/a)\int \sec^2 ax\ .(a)dx + (1/a)\int (tg\ ax)^2.(a)\sec^2 ax\ dx =$

$(1/a)tg\ ax + \dfrac{(1/a)(tg\ ax)^{2+1}}{2+1} = \dfrac{tg\ ax}{a} + \dfrac{(tg\ ax)^3}{3a} =$

$\dfrac{tg\ ax}{a} + \dfrac{tg^3\ ax}{3a} + c$.

19. $\int tg^3 \dfrac{x}{3} . \sec^3 \dfrac{x}{3} dx$

$\int tg^2 \dfrac{x}{3} . tg \dfrac{x}{3} . \sec^3 \dfrac{x}{3} dx$

Por Trigonometría: $\sec^2 \dfrac{x}{3} - 1 = tg^2 \dfrac{x}{3}$.

$\int (\sec^2 \dfrac{x}{3} - 1). tg \dfrac{x}{3} . \sec^3 \dfrac{x}{3} dx$

$\int \sec^5 \dfrac{x}{3} . tg \dfrac{x}{3} .- \int \sec^3 \dfrac{x}{3} . tg \dfrac{x}{3} . dx$

$$\int \sec^4 \frac{x}{3} \cdot \sec \frac{x}{3} \cdot \mathrm{tg}\, \frac{x}{3} \, .dx - \int \sec^2 \frac{x}{3} \cdot \sec \frac{x}{3} \cdot \mathrm{tg}\, \frac{x}{3} \cdot dx$$

$$3\int \left(\sec \frac{x}{3}\right)^4 .(1/3)\sec \frac{x}{3} .\mathrm{tg}\, \frac{x}{3} .dx - 3\int \left(\sec \frac{x}{3}\right)^2 .(1/3)\sec \frac{x}{3} .\mathrm{tg}\, \frac{x}{3} .dx$$

$$\frac{3\left(\sec \frac{x}{3}\right)^{4+1}}{4+1} - \frac{3\left(\sec \frac{x}{3}\right)^{2+1}}{2+1} = \frac{3\left(\sec \frac{x}{3}\right)^{5}}{5} - \frac{3\left(\sec \frac{x}{3}\right)^{3}}{3} =$$

$$\frac{3}{5}\left(\sec \frac{x}{3}\right)^5 - \left(\sec \frac{x}{3}\right)^3 + c \, .$$

21. $$\int \frac{dx}{\mathrm{sen}^4\, 3x \, . \cos^2 3x} \, .$$

$$\int \csc^4 3x \, . \sec^2 3x \, dx = \int \sec^2 3x \, . \csc^2 3x \, . \csc^2 3x \, .dx$$

Por Trigonometría: $\sec^2 3x = \mathrm{tg}^2\, 3x + 1$; $\csc^2 3x = 1 + \cot^2 3x$.

$$\int (\mathrm{tg}^2\, 3x + 1)(1 + \cot^2 3x).\csc^2 3x \, .dx$$

$$\int (\mathrm{tg}^2\, 3x + \mathrm{tg}^2\, 3x.\cot^2 3x + 1 + \cot^2 3x).\csc^2 3x \, .dx$$

$$\int (\mathrm{tg}^2\, 3x + \cancel{\mathrm{tg}^2\, 3x}.\frac{1}{\cancel{\mathrm{tg}^2\, 3x}} + 1 + \cot^2 3x).\csc^2 3x \, .dx$$

$$\int (\mathrm{tg}^2\, 3x + 1 + 1 + \cot^2 3x).\csc^2 3x \, .dx = \int (\mathrm{tg}^2\, 3x + 2 + \cot^2 3x).\csc^2 3x.dx$$

$$\int (\mathrm{tg}^2\, 3x.\csc^2 3x.dx + 2\int \csc^2 3x \, .dx + \int \cot^2 3x.\csc^2 3x \, .dx$$

$$\int \frac{\cancel{\mathrm{sen}^2\, 3x}}{\cos^2 3x} . \frac{1}{\cancel{\mathrm{sen}^2\, 3x}} .dx + 2\int \csc^2 3x \, .dx + \int \cot^2 3x.\csc^2 3x \, .dx$$

$$\int \frac{1}{\cos^2 3x}.dx + 2\int \csc^2 3x\ .dx + \int \cot^2 3x.\csc^2 3x\ .dx$$

$$\int \sec^2 3x\ dx + 2\int \csc^2 3x\ .dx + \int \cot^2 3x.\csc^2 3x\ .dx$$

$$\mathbf{(1/3)}\int \sec^2 3x.\mathbf{(3)}dx + 2\mathbf{(1/3)}\int \csc^2 3x.\mathbf{(3)}dx + \mathbf{(-1/3)}\int (\cot 3x)^2.\mathbf{(-3)}\csc^2 3x.\ dx$$

$$(1/3)\ tg\,3x + 2/3(-\cot 3x) + \frac{(\mathbf{-1/3})\ (\cot 3x)^{2+1}}{2+1} =$$

$$\mathbf{\frac{tg\,3x}{3} - \frac{2\cot 3x}{3} - \frac{(\cot 3x)^3}{3(3)} = \frac{tg\,3x}{3} - \frac{2\cot 3x}{3} - \frac{(\cot 3x)^3}{9} + c\ .}$$

22. $\int \left(\frac{\csc bx}{tg\ bx}\right)^2 dx$

$$\int \csc^2 bx\ .\ \cot^2 bx\ .\ dx = \int \cot^2 bx\ .\ \csc^2 bx\ .dx$$

$$(-1/b)\int (\cot bx)^2\ .(-\ b)\ \csc^2 bx\ .dx$$

$$\frac{(-1/b)(\cot bx)^{2+1}}{2+1} = -\frac{(\cot bx)^3}{3b} = -\mathbf{\frac{(\cot^3 bx)}{3b}} + \mathbf{c}\ .$$

23. $\int \left(\frac{tg\ \varphi}{\cot\ \varphi}\right)^3 d\varphi$

$$\int tg^3\ \varphi\ .tg^3\ \varphi\ .d\varphi = \int tg^2\ \varphi\ .tg\ \varphi\ .tg^2\ \varphi\ .d\varphi$$

$$\int (\sec^2 \varphi\ - 1).(\sec^2 \varphi\ - 1).\ tg^2\ \varphi\ .d\varphi = \int (\sec^2 \varphi\ - 1)^2.\ tg^2\ \varphi\ .d\varphi$$

$$\int (\sec^4 \varphi\ - 2\sec^2 \varphi\ + 1).\ tg^2\ \varphi\ .d\varphi$$

$\int (\sec^2\varphi\ .\sec^2\varphi\ - 2\sec^2\varphi\ + 1).tg^2\varphi\ .d\varphi$ .Por Trigonometría: $\sec^2\varphi\ = tg^2\varphi\ + 1$

$\int [(tg^2\varphi\ + 1).\ \sec^2\varphi\ - 2\sec^2\varphi\ + 1].\ tg^2\varphi\ .d\varphi$

$\int [(tg^2\varphi\ .\ \sec^2\varphi\ + \sec^2\varphi\ - 2\ \sec^2\varphi\ + 1].\ tg^2\varphi\ .d\varphi$

$\int [(tg^2\varphi\ .\ \sec^2\varphi\ - \sec^2\varphi\ + 1].\ tg^2\varphi\ .d\varphi$

$\int [(tg^2\varphi\ .\ tg^2\varphi\ .\ \sec^2\varphi\ - tg^2\varphi\ .\sec^2\varphi\ + tg^2\varphi\ ].\ d\varphi$

Por trigonometría: $tg^2\varphi\ = \sec^2\varphi\ - 1$.

$\int [(tg\ \varphi\ )\ .\ tg^2\varphi\ .\ \sec^2\varphi\ - tg^2\varphi\ .\sec^2\varphi\ + \sec^2\varphi\ - 1].\ d\varphi$

$\int tg^4\varphi\ .\sec^2\varphi\ .d\varphi\ - \int tg^2\varphi\ .\sec^2\varphi\ .d\varphi\ + \int \sec^2\varphi\ .d\varphi\ - \int d\varphi$

$\int (tg\ \varphi\ )^4 .\ \sec^2\varphi\ .d\varphi\ - \int (tg\ \varphi\ )^2 .\sec^2\varphi\ .d\varphi\ + \int \sec^2\varphi\ .d\varphi\ - \int d\varphi$

$\left.\begin{array}{l} v = (tg\ \varphi\ ) \\ dv = \sec^2\varphi\ .d\varphi \\ n = 4 \end{array}\right\}$ Esta completo el diferencial de la 1ra integral . Se aplica: $\int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c$.

$\left.\begin{array}{l} v = (tg\ \varphi\ ) \\ dv = \sec^2\varphi\ .d\varphi \end{array}\right\}$ Esta completo el diferencial de la 2da integral.

La 3ra y 4ta integrales,estan completos sus diferenciales, se procede a integrar .

$\dfrac{(tg\ \varphi\ )^{4+1}}{4+1} - \dfrac{(tg\ \varphi\ )^{2+1}}{2+1} + tg\ \varphi\ - \varphi$

$\dfrac{(tg\ \varphi\ )^5}{} - \dfrac{(tg\ \varphi\ )^3}{} + tg\ \varphi\ - \varphi\ = \dfrac{tg^5\varphi}{} - \dfrac{tg^3\varphi}{} + tg\ \varphi\ - \varphi$

5 3 5 3

24. $\int \left(\frac{\text{tg at}}{\cos \text{at}}\right)^4 dt$

$\int \text{tg}^4 \text{ at . sec}^4 \text{ at dt} = \int \text{tg}^4 \text{ at . sec}^2 \text{ at . sec}^2 \text{ at dt}$

$\int \text{tg}^4 \text{ at . sec}^2 \text{ at . } (1 + \text{tg}^2 \text{ at}) \text{ .dt}$

$\int (\text{tg}^4 \text{ at . sec}^2 \text{ at} + \text{tg}^4 \text{ at . sec}^2 \text{ at .tg}^2 \text{ at}) \text{ .dt}$

$\int \text{tg}^4 \text{ at . sec}^2 \text{ at . dt} + \int \text{tg}^6 \text{ at . sec}^2 \text{ at .dt}$

$\int (\text{tg at})^4 \text{ . sec}^2 \text{ at . dt} + \int (\text{tg at})^6 \text{ . sec}^2 \text{ at .dt}$

$\left.\begin{array}{l} v = (\text{tg at}) \\ dv = a.\text{sec}^2 \text{ at} \\ n = 4 \end{array}\right\}$ Falta (a) para completar el diferencial en la 1^{ra} integral. Se aplica: $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$

$\left.\begin{array}{l} v = (\text{tg at}) \\ dv = a.\text{sec}^2 \text{ at} \\ n = 6 \end{array}\right\}$ Falta (a) para completar el diferencial en la 2^{da} integral. Se aplica: $\int v^n\, dv = \frac{v^{n+1}}{n+1} + c$

$(1/a)\int (\text{tg at})^4 \text{ .(a)sec}^2 \text{ at . dt} + (1/a)\int (\text{tg at})^6 \text{ .(a)sec}^2 \text{ at .dt}$

$\frac{(1/a)\ (\text{tg at})^{4+1}}{4+1} + \frac{(1/a)\ (\text{tg at})^{6+1}}{6+1} = \frac{(\text{tg at})^5}{5a} + \frac{(\text{tg at})^7}{7a} =$

$\frac{\text{tg}^5 \text{ at}}{5a} + \frac{\text{tg}^7 \text{ at}}{7a} + c$.

25. $\int \frac{\text{tg}^3 x \ \ dx}{\sqrt{\sec x}}$

$\int \text{tg}^3 x \text{ . } (\sec x)^{-1/2} \text{ .dx} = \int \text{tg}^2 x \text{ . tg } x \ (\sec x)^{-1/2} \text{ .dx}$

$\int (\sec^2 x - 1).(\sec x)^{-1/2} . \operatorname{tg} x \, dx =$

$\int [(\sec x)^2.(\sec x)^{-1/2} - (\sec x)^{-1/2}] . \operatorname{tg} x \, dx =$

$\int [(\sec x)^{3/2} - (\sec x)^{-1/2}] . \operatorname{tg} x \, dx =$

$\int (\sec x)^{3/2} .\operatorname{tg} x \, dx - \int (\sec x)^{-1/2} .\operatorname{tg} x \, dx =$

En la $2^{\underline{da}}$ integral se hace un artificio: **$-1/2 = -3/2 + 2/2 = -3/2 + 1$.**

$\int (\sec x)^{1/2} .(\sec x)^{2/2} .\operatorname{tg} x \, dx - \int (\sec x)^{-3/2} . (\sec x)^1 .\operatorname{tg} x \, dx =$

$\int (\sec x)^{1/2}.(\sec x).\operatorname{tg} x \, dx - \int (\sec x)^{-3/2}.(\sec x).\operatorname{tg} x \, dx =$

El diferencial de ambas integrales esta completo. Se aplica: $\int v^n \, dv = \dfrac{v^{n+1}}{n+1} + c.$

$$\frac{(\sec x)^{1/2+1}}{1/2+1} - \frac{(\sec x)^{-3/2+1}}{-3/2+1} = \frac{(\sec x)^{3/2}}{3/2} - \frac{(\sec x)^{-1/2}}{-1/2} =$$

$$\frac{2(\sec x)^{3/2}}{3} + 2(\sec x)^{-1/2} = \frac{2(\sec x)^{3/2}}{3} + \frac{2}{\sqrt{\sec x}} + c .$$

26. **$\int \operatorname{tg}^n x . \sec^4 x \, dx$**

$\int \sec^2 x . \operatorname{tg}^n x . \sec^2 x \, dx = \int [(1 + \operatorname{tg}^2 x). \operatorname{tg}^n x . \sec^2 x] \, dx$

$\int \operatorname{tg}^n x . \sec^2 x . dx + \int \operatorname{tg}^2 x. \operatorname{tg}^n x . \sec^2 x .dx$

$\int (\operatorname{tg} x)^n . \sec^2 x . dx + \int (\operatorname{tg} x)^{n+2} . \sec^2 x .dx$

$v = (\operatorname{tg} x)$ $\Big\}$ El diferencial de la $1^{\underline{ra}}$ integral, esta completo.

$dv = \sec^2 x\,.dx$ Se aplica: $\int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c$
$n = n$

$v = (\text{tg } x)$
$dv = \sec^2 x\,.dx$
$n = n+2$

El diferencial de la 2da integral, esta completo.
Se aplica: $\int v^n\,dv = \dfrac{v^{n+1}}{n+1} + c$

$$\frac{(\text{tg } x)^{n+1}}{n+1} + \frac{(\text{tg } x)^{[(n+2)+1]}}{[(n+2)+1]} = \frac{(\text{tg } x)^{n+1}}{n+1} + \frac{(\text{tg } x)^{n+3}}{n+3} =$$

$$\mathbf{\frac{\text{tg}^{n+1} x}{n+1} + \frac{\text{tg}^{n+3} x}{n+3} + c}\,.$$

27. $\displaystyle\int \frac{\text{tg}^5 2\theta\,d\theta}{\sec^3 2\theta}$

$$\int \text{tg}^3 2\theta\,.\,\text{tg}^2 2\theta\,.\,\cos^3 2\theta\,.\,d\theta = \int \frac{\text{sen}^3 2\theta}{\cancel{\cos^3 2\theta}}\,.\,\text{tg}^2 2\theta\,\cancel{\cos^3 2\theta}\,.\,d\theta$$

$$\int \text{tg}^2 2\theta.\text{sen}^3 2\theta\,d\theta = \int (\sec^2 2\theta - 1).\text{sen}^2 2\theta\,.\,\text{sen } 2\theta\,d\theta$$

$$\int (\sec^2 2\theta\,.\,\text{sen}^2 2\theta\,.\,\text{sen } 2\theta - \text{sen}^2 2\theta.\,\text{sen } 2\theta)\,d\theta$$

$$\int \left(\frac{1}{\cos^2 2\theta}.\,\text{sen}^2 2\theta\,.\,\text{sen } 2\theta - (1 - \cos^2 2\theta).\,\text{sen } 2\theta\right)d\theta$$

$$\int (\text{tg}^2 2\theta\,.\,\text{sen } 2\theta - \text{sen } 2\theta + \cos^2 2\theta\,.\text{sen } 2\theta)\,d\theta$$

$$\int [(\sec^2 2\theta - 1)\,.\,\text{sen } 2\theta - \text{sen } 2\theta + \cos^2 2\theta\,.\text{sen } 2\theta)]\,d\theta$$

$$\int \sec^2 2\theta\,.\,\text{sen } 2\theta - \text{sen } 2\theta - \text{sen } 2\theta + \cos^2 2\theta\,.\text{sen } 2\theta)\,d\theta$$

$$\int \sec^2 2\theta\,.\,\text{sen } 2\theta - 2\text{sen } 2\theta + \cos^2 2\theta\,.\text{sen } 2\theta)\,d\theta$$

$$\int \frac{1}{\cos 2\theta \quad \cos 2\theta} . \operatorname{sen} 2\theta - 2\operatorname{sen} 2\theta + \cos^2 2\theta . \operatorname{sen} 2\theta)\, d\theta$$

$$\int (\operatorname{tg} 2\theta . \frac{1}{\cos 2\theta} . - 2\operatorname{sen} 2\theta + \cos^2 2\theta . \operatorname{sen} 2\theta)\, d\theta$$

$$\int (\operatorname{tg} 2\theta . \sec 2\theta . - 2\operatorname{sen} 2\theta + \cos^2 2\theta . \operatorname{sen} 2\theta)\, d\theta$$

$$\int \sec 2\theta . \operatorname{tg} 2\theta . d\theta - 2\int \operatorname{sen} 2\theta . d\theta + \int (\cos 2\theta)^2 . \operatorname{sen} 2\theta . d\theta$$

Completando los diferenciales, tenemos:

$$½\int \sec 2\theta . \operatorname{tg} 2\theta .(2)d\theta - 2(½)\int \operatorname{sen} 2\theta .(2)\, d\theta + (-½) \int (\cos 2\theta)^2 .(-2)\operatorname{sen} 2\theta . d\theta$$

$$(½)\sec 2\theta - 2(½)\,(- \cos 2\theta) - \frac{½(\cos 2\theta)^{2+1}}{2+1} =$$

$$\frac{\sec 2\theta}{2} + \cos 2\theta - \frac{(\cos 2\theta)^3}{2(3)} = \frac{\sec 2\theta}{2} + \cos 2\theta - \frac{\cos^3 2\theta}{6} + c\ .$$

Problemas. Páginas 265

Demostrar las siguientes Integraciones:

1. $\int \mathbf{sen^2\, x\, .\, dx} = \dfrac{\mathbf{x}}{\mathbf{2}} - \dfrac{\mathbf{sen\ 2x}}{\mathbf{4}} + \mathbf{c}$.

$\int (½ - ½ \cos 2x)\, dx = ½ \int dx - ½\, .\, ½ \int \cos 2x\, .\, dx =$

$x - ¼ \operatorname{sen} 2x = \dfrac{x}{2} - \dfrac{\operatorname{sen} 2x}{4} + c$.

2. $\int \mathbf{sen^4\, x\, .\, dx} = \dfrac{\mathbf{3x}}{\mathbf{8}} - \dfrac{\mathbf{sen\ 2x}}{\mathbf{4}} + \dfrac{\mathbf{sen\ 4x}}{\mathbf{32}} + \mathbf{c}$.

$\int \operatorname{sen}^2 x\, .\, \operatorname{sen}^2 x\, dx = \int (½ - ½ \cos 2x)^2\, dx =$

$\int \{(½)^2 - 2(½)(½) \cos 2x + [(½) \cos 2x]^2\}\, dx$

$\int \{¼ - ½ \cos 2x + ¼ \cos^2 2x\}\, dx$

$¼ \int dx - ½ \int \cos 2x\, dx + ¼ \int \cos^2 2x\, dx$

$¼ \int dx - ½.½ \int \cos 2x\, .(2)dx + ¼ \int [½ + ½ \cos 2(2x)]\, dx$

$¼ \int dx - ¼ \int \cos 2x\, .(2)dx + ¼ \int [½ + ½ \cos 4x]\, dx$

$¼ \int dx - ¼ \int \cos 2x\, .(2)dx + ¼.½ \int dx + ¼.½ \int \cos 4x\, dx$

$\frac{1}{4}\int dx - \frac{1}{4}\int \cos 2x\,.(2)dx + \frac{1}{4}.\frac{1}{2}\int dx + \frac{1}{4}.\frac{1}{2}.\frac{1}{4}\int \cos 4x\ (4)dx$

$\frac{1}{4}\int dx - \frac{1}{4}\int \cos 2x\,.(2)dx + \frac{1}{8}\int dx + 1/32\int \cos 4x\,.(4)dx$

$\frac{1}{4} x - \frac{1}{4}\,\text{sen}\ 2x + \frac{1}{8} x + 1/32\ \text{sen}\ 4x =$
$\frac{1}{4} x + \frac{1}{8} x - \frac{1}{4}\,\text{sen}\ 2x + 1/32\ \text{sen}\ 4x =$
$2/8\ x + \frac{1}{8} x - \frac{1}{4}\,\text{sen}\ 2x + 1/32\ \text{sen}\ 4x =$

$\mathbf{\frac{3}{8} x - \frac{1}{4}\,\text{sen}\ 2x + 1/32\ \text{sen}\ 4x + c}$.

3. $\int \cos^4 x\, dx = \dfrac{3x}{8} + \dfrac{\text{sen}\ 2x}{4} + \dfrac{\text{sen}\ 4x}{32} + c$.

$\int \cos^2 x.\ \cos^2 x\ dx = \int [\frac{1}{2} + \frac{1}{2}\cos 2x]^2\ dx\ dx =$

$\int \{(\frac{1}{2})^2 + 2(\frac{1}{2})(\frac{1}{2})\cos 2x + [(\frac{1}{2})\cos 2x]^2\}\ dx$

$\int \{\frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x\}\ dx$

$\frac{1}{4}\int dx + \frac{1}{2}\int \cos 2x\ dx + \frac{1}{4}\int \cos^2 2x\ dx$

$\frac{1}{4}\int dx + \frac{1}{2}.\frac{1}{2}\int \cos 2x\ (2)dx + \frac{1}{4}\int [\frac{1}{2} + \frac{1}{2}\cos 2(2)x]\ dx$

$\frac{1}{4}\int dx + \frac{1}{4}\int \cos 2x\ (2)dx + \frac{1}{4}\int [\frac{1}{2} + \frac{1}{2}\cos 4x]\ dx$

$\frac{1}{4}\int dx + \frac{1}{4}\int \cos 2x\ (2)dx + \frac{1}{4}.\frac{1}{2}\int dx + \frac{1}{4}.\frac{1}{2}\int \cos 4x\ dx$

$\frac{1}{4}\int dx + \frac{1}{4}\int \cos 2x\ (2)dx + \frac{1}{8}\int dx + \frac{1}{4}.\frac{1}{2}.\frac{1}{4}\int \cos 4x\ (4)dx$

$\frac{1}{4}\int dx + \frac{1}{4}\int \cos 2x\ (2)dx + \frac{1}{8}\int dx + 1/32\int \cos 4x\ (4)dx$

$\frac{1}{4} x + \frac{1}{4}\,\text{sen}\ 2x + \frac{1}{8} x + 1/32\ \text{sen}\ 4x =$

$\frac{1}{4} x + \frac{1}{8} x + \frac{1}{4}$ sen $2x + 1/32$ sen $4x$ =

$2/8\, x + \frac{1}{8} x + \frac{1}{4}$ sen $2x + 1/32$ sen $4x$ =

$3/8\, x + \frac{1}{4}$ **sen** $2x + 1/32$ **sen** $4x + c$.

4. $\int \text{sen}^6 x\, dx = \dfrac{5x}{16} - \dfrac{\text{sen } 2x}{4} + \dfrac{\text{sen}^3 2x}{48} + \dfrac{3\text{sen } 4x}{64} + c$.

$\int \text{sen}^2 x \,.\, \text{sen}^2 x \,.\, \text{sen}^2 x\, dx = \int (\text{sen}^2 x)^3 dx =$

$\int [\frac{1}{2} - \frac{1}{2} \cos 2x]^3\, dx$

$\int \{(\frac{1}{2})^3 - 3.(\frac{1}{2})^2.\ \frac{1}{2} \cos 2x + 3(\frac{1}{2}).\ (\frac{1}{2} \cos 2x)^2 - (\frac{1}{2} \cos 2x)^3\}\, dx$

$\int (\frac{1}{8} - 3.\frac{1}{4}.\frac{1}{2} \cos 2x + 3(\frac{1}{2}).(\frac{1}{2})^2.\cos^2 2x - \frac{1}{8}\cos^3 2x)\, dx$

$\int (\frac{1}{8} - \frac{3}{8}\cos 2x + 3(\frac{1}{2}).(\frac{1}{4})\cos^2 2x - \frac{1}{8}\cos^3 2x)\, dx$

$\int (\frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos^3 2x)\, dx$

$\int \{\frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{8}[\frac{1}{2} + \frac{1}{2} \cos (2)2x] - \frac{1}{8}.\cos^2 2x.\ \cos 2x\}\, dx$

$\int \{\frac{1}{8} - \frac{3}{8} \cos 2x + 3/16 + 3/16 \cos 4x - \frac{1}{8}.[(1 - \text{sen}^2 2x).\ \cos 2x]\}\, dx$

$\int \{\frac{1}{8} + 3/16 - \frac{3}{8} \cos 2x + 3/16 \cos 4x - \frac{1}{8}[\cos 2x - \text{sen}^2 2x.\cos 2x]\}dx$

$\int \{2/16 + 3/16 - \frac{3}{8} \cos 2x + 3/16 \cos 4x - \frac{1}{8}\cos 2x + \frac{1}{8}\text{sen}^2 2x.\cos\ 2x\}dx$

$\int \{5/16 - \frac{3}{8} \cos 2x - \frac{1}{8}\cos 2x + 3/16 \cos 4x + \frac{1}{8}(\text{sen } 2x)^2.\cos 2x\}dx$

$\int \{5/16 - 4/8 \cos 2x + 3/16 \cos 4x + ⅛(\text{sen } 2x)^2.\cos 2x\}dx$

$\int \{5/16 - ½ \cos 2x + 3/16 \cos 4x + ⅛(\text{sen } 2x)^2.\cos 2x\}dx$

$5/16\int dx - ½\int \cos 2x .(2)dx + 3/16\int \cos 4x .(4)dx + ⅛\int (\text{sen}2x)^2.\cos 2x\ dx$

$5/16\int dx - ½.½\int \cos 2x .(2)dx + 3/16.¼\int \cos 4x .(4)dx + ⅛.½\int (\text{sen}2x)^2.\ (2)\cos 2x dx$

$5/16\int dx - ¼\int \cos 2x .(2)dx + 3/64\int \cos 4x .(4)dx + 1/16\int (\text{sen}2x)^2.(2)\cos 2x dx$

$5/16\ x - ¼ \text{ sen } 2x + 3/64 \text{ sen } 4x + \frac{1/16(\text{sen}^3 2x)}{3} =$

$5/16\ x - ¼ \text{ sen } 2x + 3/64 \text{ sen } 4x + 1/48\ (\text{sen}^3 2x) =$

$\mathbf{\frac{5x}{16} - \frac{\text{sen } 2x}{4} + \frac{\text{sen}^3 2x}{48} + \frac{3\text{sen } 4x}{64} + c}$.

5. $\int \cos^6 x\ dx = \frac{5x}{16} + \frac{\text{sen } 2x}{4} - \frac{\text{sen}^3 2x}{48} + \frac{3\text{sen } 4x}{64} + c$.

$\int \cos^2 x.\ \cos^2 x.\ \cos^2 x\ dx = \int (\cos^2 x)^3\ dx = \int [½ + ½ \cos 2x]^3\ dx$

$\int [(½)^3 + 3.(½)^2\ (½ \cos 2x) + 3(½).(½ \cos 2x)^2 + (½ \cos 2x)^3\ ]^3$
dx

$\int (⅛ + 3.¼.½ \cos 2x + 3(½).(½)^2.\cos^2 2x + ⅛\cos^3 2x)\ dx$

$\int (⅛ + ⅜\cos 2x + 3(½).(¼)\cos^2 2x + ⅛\cos^3 2x)\ dx$

$\int (⅛ + ⅜\cos 2x + ⅜ \cos^2 2x + ⅛\cos^3 2x)\ dx$

$\int \{⅛ + ⅜\cos 2x + ⅜[½ + ½ \cos (2)2x] + ⅛.\cos^2 2x.\ \cos 2x\}$
dx

$\int \{1/8 + 3/8 \cos 2x + 3/16 + 3/16 \cos 4x + 1/8[(1 - sen^2 2x).\cos 2x]\} dx$

$\int \{1/8 + 3/16 + 3/8 \cos 2x + 3/16 \cos 4x + 1/8[\cos 2x - sen^2 2x.\cos 2x]\}dx$

$\int \{2/16 + 3/16 + 3/8\cos 2x + 3/16 \cos 4x + 1/8\cos 2x - 1/8 sen^2 2x.\cos 2x\}dx$

$\int \{5/16 + 3/8 \cos 2x + 1/8\cos 2x + 3/16 \cos 4x - 1/8(sen\ 2x)^2.\cos 2x\}dx$

$\int \{5/16 + 4/8 \cos 2x + 3/16 \cos 4x - 1/8(sen\ 2x)^2.\cos 2x\}dx$

$\int \{5/16 + 1/2 \cos 2x + 3/16 \cos 4x - 1/8(sen\ 2x)^2.\cos 2x\}dx$

$5/16\int dx + 1/2 \int \cos 2x .(2)dx + 3/16 \int \cos 4x .(4)dx - 1/8\int (sen 2x)^2.\cos 2x\ dx$

$5/16\int dx + 1/2.1/2 \int \cos 2x.(2)dx + 3/16.1/4 \int \cos 4x .(4)dx - 1/8 .1/2\int (sen 2x)^2. (2)\cos 2x dx$

$5/16\int dx + 1/4 \int \cos 2x .(2)dx + 3/64 \int \cos 4x .(4)dx - 1/16\int (sen 2x)^2. (2)\cos 2x dx$

$5/16\ x + 1/4\ sen\ 2x + 3/64\ sen\ 4x - \frac{1/16(sen\ 2x)^{2+1}}{2+1} =$

$5/16\ x + 1/4\ sen\ 2x + 3/64\ sen\ 4x - \frac{1/16\ (sen\ 2x)^3}{3} =$

$\frac{5x}{16} + \frac{sen\ 2x}{4} - \frac{sen^3\ 2x}{48} + \frac{3sen\ 4x}{64} + c$.

6. $\int sen^2\ ax\ dx = \frac{x}{2} - \frac{sen\ 2ax}{4a} + c$.

$\int [\frac{1}{2} - \frac{1}{2} \cos 2ax]dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2ax]dx =$

$\frac{1}{2} \int dx - \frac{1}{2}.1/2a \int \cos 2ax \,.\, (2a)]dx =$

$\frac{1}{2} x - 1/4a\ .\text{sen } 2ax = \frac{x}{2} - \frac{\text{sen } 2ax}{4a} + c$.

7. $\int \text{sen}^2 x/2 \,.\, \cos^2 x/2\ dx = \frac{x}{8} - \frac{\text{sen } 2x}{16} + c$.

$\int [\frac{1}{2} - \frac{1}{2} \cos 2(x/2)].[\frac{1}{2} + \frac{1}{2} \cos 2(x/2)]dx$. Simplificando:

$\int [\frac{1}{2} - \frac{1}{2} \cos x].[\frac{1}{2} + \frac{1}{2} \cos x]dx$. Tenemos una diferencia de cuadrados.

$\int \{[\frac{1}{2}]^2 - [\frac{1}{2} \cos x]^2\}\ dx = \int \{[\frac{1}{4}] - [\frac{1}{4} \cos^2 x]\}\ dx =$

$\frac{1}{4}\int dx - \frac{1}{4} \int \cos^2 x\ dx = \frac{1}{4}\int dx - \frac{1}{4}\ \{\int [\frac{1}{2} + \frac{1}{2} \cos 2x]\ dx\}$

$\frac{1}{4}\int dx - \frac{1}{4}.\frac{1}{2} \int dx - \frac{1}{4}.\frac{1}{2} \int \cos 2x\ dx = \frac{1}{4}\int dx - \frac{1}{8} \int dx - \frac{1}{8} \int \cos 2x\ dx$

$\frac{1}{4}\int dx - \frac{1}{8} \int dx - \frac{1}{8}.(\frac{1}{2}) \int \cos 2x\ .(2)\ dx =$

$\frac{1}{4}\int dx - \frac{1}{8} \int dx - 1/16 \int \cos 2x\ .(2)\ dx$

$\frac{1}{4} x - \frac{1}{8} x - 1/16\ \text{sen } 2x = 2/8\, x - \frac{1}{8} x - 1/16\ \text{sen } 2x =$

$1/8\, x - 1/16\ \text{sen } 2x = \frac{x}{8} - \frac{\text{sen } 2x}{16} + c$.

8. $\int \text{sen}^4 ax\ dx$

$\int sen^2\, ax\,.\, sen^2\, ax\, dx = \int sen^2\, ax\,.\, sen^2\, ax\, dx = \int [½ - ½ \cos 2ax]^2 dx$

$\int \{(½)^2 - 2(½)(½).\cos 2ax + [(½)\cos 2ax]^2\}\, dx$

$\int \{¼ - ½ \cos 2ax + ¼ \cos^2 2ax\}\, dx$
$¼ \int dx - ½ \int \cos 2ax\, dx + ¼ \int \cos^2 2ax\, dx$

$¼ \int dx - ½.½a \int \cos 2ax\, (2a)dx + ¼ \int [½ + ½ \cos 2(2ax)]\, dx$

$¼ \int dx - ¼a \int \cos 2ax\, (2a)dx + ¼ \int [½ + ½ \cos 4ax]\, dx$

$¼ \int dx - ¼a \int \cos 2ax\, (2a)dx + ¼.½ \int dx + ¼.½ \int \cos 4ax\, dx$

$¼ \int dx - ¼a \int \cos 2ax\, (2a)dx + ¼.½ \int dx + ¼.½.¼a \int \cos 4ax\, (4a)dx$

$¼ \int dx - ¼a \int \cos 2ax\, (2a)dx + ⅛ \int dx + 1/32a \int \cos 4ax\, (4a)dx$

$¼\, x - ¼a\, sen\, 2ax + ⅛\, x + 1/32a\, sen\, 4ax =$
$¼\, x + ⅛\, x - ¼a\, sen\, 2ax + 1/32a\, sen\, 4ax =$
$2/8\, x + ⅛\, x - ¼a\, sen\, 2ax + 1/32a\, sen\, 4ax =$

$⅜\, x - ¼a\, sen\, 2ax + 1/32a\, sen\, 4ax =$

$$\mathbf{⅜\, x - \frac{sen\, 2ax}{4a} + \frac{sen\, 4ax}{32a} + c\,.}$$

9. $\int \mathbf{sen^2\, 2x\, .\cos^4 2x\, dx}$

$\int sen^2\, 2x\, .\cos^4 2x\, .\, \cos^2 2x\, dx$

$\int [½ - ½ \cos 2(2x)].\, [½ + ½ \cos 2(2x)].\, \cos^2 2x\, dx$

$\int [½ - ½ \cos 4x]. [½ + ½ \cos 4x]. \cos^2 2x\, dx$

$\int [¼ - ¼ \cos^2 4x].\cos^2 2x\, dx$

$\int [¼ .\cos^2 2x - ¼.\cos^2 2x .\cos^2 4x]\, dx$

$\int \{¼[½ + ½ \cos 2(2)x] - ¼.\cos^2 2x\, (1 - \text{sen}^2 4x)\} . dx$

$\int \{¼[½ + ½ \cos 4x] - ¼ \cos^2 2x + ¼ \text{sen}^2 4x.\cos^2 2x\} . dx$

$\int \{⅛ + ⅛ \cos 4x - ¼ [½ + ½ \cos 2(2)x] + ¼ \text{sen}^2 4x[½ + ½ \cos 2(2)x]\}.dx$

$\int \{⅛ + ⅛ \cos 4x - ¼ [½ + ½ \cos 4x] + ¼ \text{sen}^2 4x[½ + ½ \cos 4x]\}.dx$

$\int \{⅛ + ⅛ \cos 4x - ⅛ - ⅛ \cos 4x + ⅛ \text{sen}^2 4x + ⅛ \text{sen}^2 4x .\cos 4x]\}.dx$

$\int \{\cancel{⅛} + \cancel{⅛ \cos 4x} - \cancel{⅛} - \cancel{⅛ \cos 4x} + ⅛ \text{sen}^2 4x + ⅛ \text{sen}^2 4x .\cos 4x]\}.dx$

$\int [⅛ \text{sen}^2 4x + ⅛ \text{sen}^2 4x .\cos 4x].dx$

$⅛ \int [½ - ½ \cos 2(4)x]dx + ⅛ \int (\text{sen } 4x)^2.\cos 4x.dx$

$⅛ \int [½ - ½ \cos 8x]dx + ⅛.¼ \int (\text{sen } 4x)^2 .\cos 4x.(4)dx$

$⅛ .½ \int dx - ⅛.½ \int \cos 8x\, dx + 1/32 \int (\text{sen } 4x)^2 .\cos 4x.(4)dx$

$1/16 \int dx - 1/16.⅛ \int \cos 8x .(8)dx + 1/32 \int (\text{sen } 4x)^2 .\cos 4x.(4)dx$

$1/16\, x - 1/128 \text{ sen } 8x + \dfrac{1/32\, (\text{sen } 4x)^{2+1}}{2+1} =$

$$\frac{x}{16} - \frac{\text{sen } 8x}{128} + \frac{(\text{sen } 4x)^3}{32(3)} = \frac{x}{16} + \frac{(\text{sen } 4x)^3}{96} - \frac{\text{sen } 8x}{128} =$$

$$\frac{x}{16} + \frac{\text{sen}^3 4x}{96} - \frac{\text{sen } 8x}{128} + c .$$

10. $\int (2 - \text{sen } \theta)^2 \, d\theta = \frac{9\theta}{2} + 4\cos\theta + \frac{\text{sen } 2\theta}{4} + c .$

$$\int [4 - 2.2.\text{sen } \theta + (\text{sen } \theta)^2] \, d\theta = \int [4 - 4\text{sen } \theta + \text{sen}^2 \theta] \, d\theta =$$

$$\int \{4 - 4\text{sen } \theta + [\tfrac{1}{2} + \tfrac{1}{2} \cos 2\theta]\}d\theta = \int \{4 - 4\text{sen } \theta + \tfrac{1}{2} + \tfrac{1}{2} \cos 2\theta]\}d\theta$$

$$\int \{8/2 + \tfrac{1}{2} - 4\text{sen } \theta + \tfrac{1}{2} \cos 2\theta]\}d\theta = \int \{9/2 - 4\text{sen } \theta + \tfrac{1}{2} \cos 2\theta]\}d\theta$$

$$9/2 \int d\theta - 4\int \text{sen } \theta \, .d\theta + \tfrac{1}{2} \int \cos 2\theta \, .d\theta =$$

$$9/2 \int d\theta - 4\int \text{sen } \theta \, .d\theta + \tfrac{1}{2}.\tfrac{1}{2} \int \cos 2\theta \, .(2)d\theta =$$

$$9/2\, \theta - 4(- \cos \theta) + \tfrac{1}{4} (\text{sen } 2\theta) = 9/2\, \theta + 4\cos \theta + \tfrac{1}{4} (\text{sen } 2\theta)$$

$$\frac{9\theta}{2} + 4\cos\theta + \frac{\text{sen } 2\theta}{4} + c .$$

11. $\int [\text{sen}^2 \Phi + \cos \Phi]^2 \, d\Phi =$

$$\int [(\text{sen}^2 \Phi)^2 + 2.(\text{sen}^2 \Phi).\cos \Phi + \cos^2 \Phi]^2 \, d\Phi =$$

$$\int [(\tfrac{1}{2} - \tfrac{1}{2} \cos 2\Phi)^2 + 2.(\text{sen}^2 \Phi).\cos \Phi + (\tfrac{1}{2} + \tfrac{1}{2} \cos 2\Phi)] \, d\Phi =$$

$$\int [(\tfrac{1}{4} - 2.\tfrac{1}{2}.\, \tfrac{1}{2} \cos 2\Phi + (\tfrac{1}{2} \cos 2\Phi)^2 + 2(\text{sen}^2 \Phi).\cos \Phi + \tfrac{1}{2} + \tfrac{1}{2} \cos 2\Phi]d\Phi =$$

$$\int [(\tfrac{1}{4} - \tfrac{1}{2} \cos 2\Phi + \tfrac{1}{4} \cos^2 2\Phi + 2\text{sen}^2 \Phi.\cos \Phi + \tfrac{1}{2} + \tfrac{1}{2} \cos 2\Phi]d\Phi =$$

$\int [(\frac{1}{4} - \cancel{\frac{1}{2}\cos 2\Phi} + \frac{1}{4}\cos^2 2\Phi + 2\operatorname{sen}^2 \Phi.\cos\Phi + \frac{1}{2} + \cancel{\frac{1}{2}\cos}\ 2\Phi]d\,\Phi =$

$\int [(\frac{1}{4} + \frac{1}{2} + \frac{1}{4}\cos^2 2\Phi + 2\operatorname{sen}^2 \Phi.\cos\Phi]d\,\Phi =$

$\int [(\frac{1}{4} + \frac{1}{2} + \frac{1}{4}(\frac{1}{2} + \frac{1}{2}\cos 2(2\Phi) + 2\operatorname{sen}^2 \Phi.\cos\Phi]d\,\Phi =$

$\int [(\frac{1}{4} + \frac{1}{2} + \frac{1}{8} + \frac{1}{8}\cos 4\Phi + 2\operatorname{sen}^2 \Phi.\cos\Phi]d\,\Phi =$

$\int [(2/8 + 4/8 + \frac{1}{8} + \frac{1}{8}\cos 4\Phi + 2\operatorname{sen}^2 \Phi.\cos\Phi]d\,\Phi =$

$\int [(7/8 + \frac{1}{8}\cos 4\Phi + 2\operatorname{sen}^2 \Phi.\cos\Phi]d\,\Phi =$

$7/8\int d\Phi + \frac{1}{8}\int \cos 4\Phi + 2\int (\operatorname{sen}\Phi)^2.\cos\Phi.d\,\Phi =$

$7/8\int d\Phi + \frac{1}{8}.\frac{1}{4}\int \cos 4\Phi\ .(4\,\Phi).d\,\Phi + 2\int (\operatorname{sen}\Phi)^2.\cos\Phi.d\,\Phi =$

$7/8\,\Phi + 1/32\ \operatorname{sen} 4\Phi + \dfrac{2(\operatorname{sen}\Phi)^{2+1}}{2+1} =$

$\dfrac{7\,\Phi}{8} + \dfrac{\operatorname{sen} 4\Phi}{32} + \dfrac{2(\operatorname{sen}\Phi)^3}{3} = \dfrac{7\,\Phi}{8} + \dfrac{2\operatorname{sen}^3\Phi}{3} + \dfrac{\operatorname{sen} 4\Phi}{32} + c$.

12. $\int \operatorname{sen} 2x \cos 4x\, dx = \dfrac{\cos 2x}{4} - \dfrac{\cos 6x}{12} + c$.

Por Trigonometría: $\operatorname{sen} 2x \cos 4x = \frac{1}{2}\operatorname{sen}[2+4]x + \frac{1}{2}\operatorname{sen}[2-4]x$

$\operatorname{sen} 2x \cos 4x = \frac{1}{2}\operatorname{sen} 6x + \frac{1}{2}\operatorname{sen}[-2]x$

$\int \{\frac{1}{2}\operatorname{sen} 6x + \frac{1}{2}\operatorname{sen}[-2]x\}dx = \int \{\frac{1}{2}\operatorname{sen} 6x - \frac{1}{2}\operatorname{sen} 2x\}dx$

$\frac{1}{2}\int \operatorname{sen} 6x\ .dx - \frac{1}{2}\int \operatorname{sen} 2x\ .dx = \frac{1}{2}.1/6\int \operatorname{sen} 6x.(6)dx - \frac{1}{2}.\frac{1}{2}\int \operatorname{sen} 2x\ .(2)dx$

$1/12\,(-\cos 6x) - \frac{1}{4}\,(-\cos 2x) = \dfrac{-\cos 6x}{12} + \dfrac{\cos 2x}{4} =$

$\dfrac{\cos 2x}{4} - \dfrac{\cos 6x}{12} + c\,.$

13. $\int \operatorname{sen} 3x \operatorname{sen} 2x\,dx = \dfrac{\operatorname{sen} x}{2} - \dfrac{\operatorname{sen} 5x}{10} + c\,.$

Por Trigonometría: $\operatorname{sen} 3x \operatorname{sen} 2x = -\frac{1}{2}\cos[3+2]x + \frac{1}{2}\cos[3-2]x$

$\operatorname{sen} 3x \operatorname{sen} 2x = -\frac{1}{2}\cos 5x + \frac{1}{2}\cos x$

$\int [-\frac{1}{2}\cos 5x + \frac{1}{2}\cos x]\,dx = -\frac{1}{2}\int \cos 5x\,.\,dx + \frac{1}{2}\int \cos x\,.\,dx =$

$-\frac{1}{2}.(1/5)\int \cos 5x\,.(5)\,dx + \frac{1}{2}\int \cos x\,.dx = -(1/10)\operatorname{sen} 5x + \frac{1}{2}\operatorname{sen} x$

$=$

$\frac{1}{2}\operatorname{sen} x - (1/10)\operatorname{sen} 5x = \dfrac{\operatorname{sen} x}{2} - \dfrac{\operatorname{sen} 5x}{10} + c\,.$

14. $\int \cos 4x \cos 3x\,dx$

Por Trigonometría: $\cos 4x \cos 3x = \frac{1}{2}\cos[4+3]x + \frac{1}{2}\cos[4-3]x$

$\cos 4x \cos 3x = \frac{1}{2}\cos 7x + \frac{1}{2}\cos x$

$\int (\frac{1}{2}\cos 7x + \frac{1}{2}\cos x)\,dx = \frac{1}{2}\int \cos 7x\,dx + \frac{1}{2}\int \cos x\,dx$

$\frac{1}{2}.(1/7)\int \cos 7x\,.(7)dx + \frac{1}{2}\int \cos x\,dx$

$1/14(\operatorname{sen} 7x) + \frac{1}{2}(\operatorname{sen} x) = \dfrac{\operatorname{sen} 7x}{14} + \dfrac{\operatorname{sen} x}{2} = \dfrac{\operatorname{sen} x}{14} + \dfrac{\operatorname{sen} 7x}{2} + c$

15. $\int \cos^2 ax\, dx =$

$\int [\ ½ + ½ \cos\ 2(ax)\ ]dx = \int [\ ½ + ½ \cos\ 2ax\ ]dx =$

$½ \int dx + ½\ .1/2a \int \cos\ 2ax\ .(2a)\ ]dx =$

$x/2 + 1/4a\ \ \text{sen}\ 2ax = x/2 + \text{sen}\ 2ax\ /4a + c\ .$

16. $\int \cos^4 ax\, dx =$

$\int \cos^2 ax\ .\ \cos^2 ax\ .dx = \int [\ ½ + ½ \cos\ 2ax]\ [½ + ½ \cos\ 2ax]\ dx =$

$\int [½ + ½ \cos\ 2ax]^2\ dx = \int [¼ + 2.½.½ \cos\ 2ax + ¼ \cos^2 2ax]dx$
$=$

$\int \{¼ + ½ \cos\ 2ax + ¼\ [½ + ½ \cos\ 2(2ax)]\}dx =$

$\int \{¼ + ½ \cos\ 2ax + ⅛ + ⅛ \cos\ 4ax\}dx$ .Haciendo operaciones:

$\int \{⅜ + ½ \cos\ 2ax + ⅛ \cos\ 4ax\}dx$

$⅜ \int dx + ½\ .1/2a \int \cos\ 2ax\ .(2a)\ dx + ⅛.¼a \int \cos\ 4ax\ .(4a)\}dx$

$3x/8 + 1/4a\ \text{sen}\ 2ax + 1/32a\ \text{sen} 4ax + c\ .$

17. $\int \text{sen}^2\ ax\ .\ \cos^2 ax\ .dx =$

$\int [\ ½ - ½ \cos\ 2(ax)]\ [½ + ½ \cos\ 2(ax)]\ dx =$

$\int [(½)^2 - (½ \cos\ 2ax)^2]\ dx = \int [¼ - ¼\ [½ + ½ \cos\ 2(2ax)^2]\ dx =$

$\int [¼ - ⅛ - ⅛ \cos\ 4ax]\ dx = \int [2/8 - ⅛ - ⅛ \cos\ 4ax]\ dx =$

$\int [\frac{1}{8} - \frac{1}{8}\cos 4ax]\, dx = \frac{1}{8}\int dx - \frac{1}{8}.\frac{1}{4a}\int \cos 4ax.(4a)]\, dx =$

$x/8 - 1/32a\,.\text{sen } 4ax = x/8 - \text{sen } 4ax/32a + c$.

18. $\int \text{sen}^4 \theta/2 \cos^2 \theta/2\, .d\theta =$

$\int \text{sen}^2 \theta/2.\cos^2 \theta/2.\text{sen}^2 \theta/2\, d\theta = \int (\text{sen } \theta/2\, .\cos \theta/2)^2\, .\text{sen}^2 \theta/2\, d\theta =$

Por Trigonometría:$\text{sen } 2x = 2\text{sen}x.\cos x$; $\text{sen } \theta/2\, .\cos \theta/2 = \text{sen}(2.\theta/2)$

$\text{sen } \theta/2\, .\cos \theta/2 = \frac{1}{2} \text{sen } \theta$.
$\int (\text{sen } \theta/2\, .\cos \theta/2)^2\, .\text{sen}^2 \theta/2\, d\theta =$

$\int (\frac{1}{2}\text{sen } \theta.\ \text{sen}^2\theta/2)\, d\theta = \int \{(\frac{1}{2}\text{sen } \theta\, [\frac{1}{2} - \frac{1}{2}\cos (2.\theta/2)]\}\, d\theta =$
$\int [\frac{1}{2}\text{sen } \theta\, (\frac{1}{2} - \frac{1}{2}\cos \theta)]\, d\theta = \int [\frac{1}{4}\, \text{sen } \theta - \frac{1}{4}\, \text{sen } \theta\, .\cos \theta]\, d\theta =$

$\frac{1}{4} \int \text{sen } \theta\, d\theta - \frac{1}{4} \int (\text{sen } \theta)^1\, .\cos \theta\, d\theta$.

$$\left.\begin{array}{l} v = \text{sen } \theta \\ dv = \cos \theta\, d\theta \\ n = 1 \end{array}\right\}$$

El diferencial esta completo, se procede a integrar.
Se usa: $\int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c$.

$$\frac{1}{4}\, (- \cos \theta) - \frac{1}{4}\, .\frac{\text{sen}^2 \theta}{2} = - \frac{\cos \theta}{4} - \frac{\text{sen}^2 \theta}{8} + c .$$

19. $\int \left(\dfrac{\csc ax}{\cot ax}\right)^4 .\ dx =$

Por Trigonometría: $\dfrac{\csc ax}{\cot ax} = \dfrac{\frac{1}{\text{sen } ax}}{\frac{\cos ax}{\text{sen } ax}} = \dfrac{1}{\cos ax} = \sec ax$

$\int \left(\frac{\csc ax}{\cot ax} \right)^4 . dx = \int (\sec ax)^4 . dx = \int \sec^4 ax . dx = \int \sec^2 ax . \sec^2 ax \, dx$

$=$

$\int (1 + tg^2 ax) . \sec^2 ax . dx = \int (\sec^2 ax + tg^2 ax . \sec^2 ax) . dx =$

$\int \sec^2 ax . dx + \int tg^2 ax . \sec^2 ax) . dx =$

$\int \sec^2 ax . dx + \int (tg \, ax)^2 . \sec^2 ax . dx =$

$v = ax$ — 1ra integral : Falta (a) para completar el diferencial.
$dv = a \, dx$ — Se aplica: $\int \sec^2 v = tg \, v + c$.

$v = tg \, ax$ — 2da integral : Falta (a) para completar el diferencial.
$dv = a . \sec^2 ax \, dx$ — Se aplica: $\int v^n dv = \frac{v^{n+1}}{n+1} + c$.

$\frac{1}{a} \int \sec^2 ax . (a) \, dx + \frac{1}{a} . \int (tg \, ax)^2 . (a) \sec^2 ax . dx =$

$\frac{tg \, ax}{a} + \frac{(tg \, ax)^{2+1}}{(2+1)a} = \frac{tg \, ax}{a} + \frac{(tg \, ax)^{2+1}}{3a} + c$.

20. $\int \mathbf{sen^2 \, x . cos^6 \, x . dx}$.

$\int sen^2 x . \cos^2 x . \cos^2 x . \cos^2 x . dx$.

$\int (sen \, x . \cos x)^2 . (\cos x . \cos x)^2 . dx$.

Por trigonometría: $sen \, x . \cos x = \frac{sen \, 2x}{2}$;

$\cos^2 x = \frac{\cos 2x + 1}{2} = ½ \cos 2x + ½$ .Sustituyendo en la integral .

$\int$ (sen x . cos x)^2 .(cos x . cos x)^2 . dx .
$\int$ (½ sen 2x)^2 .(½ cos 2x + ½)^2 . dx .
$\int$ (½ sen 2x)^2 .(½ cos 2x + ½)^2 . dx .Haciendo operaciones.
$\int$ (1/4 sen^2 2x) .[1/4 cos^2 2x + 2. ½ cos 2x . ½ + ¼] . dx .
$\int$ (1/4 sen^2 2x) .[1/4 cos^2 2x + ½ cos 2x + ¼] . dx .
$\int$ [1/16 sen^2 2x . cos^2 2x + 1/8 sen^2 2x . cos 2x + 1/16 sen^2 2x].dx .
1/16$\int$ sen^2 2x . cos^2 2x + 1/8 . ½ $\int$ (sen 2x)^2.cos 2x.(2)+ 1/16$\int$ sen^2 2x.dx .

1/16$\int$ sen^2 2x.cos^2 2x +1/16 $\int$ (sen 2x)^2.cos 2x.(2)+ 1/16$\int$ [1/2 - ½ cos 2(2x)].dx .

1/16$\int$ (sen 2x .cos 2x)^2 dx + $\frac{(\text{sen } 2x)^{2+1}}{16(2+1)}$ + 1/16.1/2 $\int$ dx - 1/16 . ½ .1/4 $\int$ cos 4x .(4)dx .

1/16$\int$ [1/2sen 2(2x)]^2 dx + $\frac{(\text{sen}^3\, 2x)}{16(3)}$ + 1/32 x - 1/128 $\int$ cos 4x .(4)dx .

1/16$\int$ [1/4sen^2 4x)] dx + $\frac{(\text{sen}^3\, 2x)}{48}$ + 1/32 x - 1/128 sen 4x .

1/16 . 1/4 $\int$ [sen^2 4x]. dx + $\frac{(\text{sen}^3\, 2x)}{48}$ + 1/32 x - 1/128 sen 4x .

1/16 . 1/4 $\int$ [1/2 – ½ cos 2(4x)]dx + $\frac{(\text{sen}^3\, 2x)}{48}$ + 1/32 x - 1/128 sen 4x .

1/64 $\int$ [1/2 – ½ cos 8x]dx + $\frac{(\text{sen}^3\, 2x)}{48}$ + 1/32 x - 1/128 sen 4x .

1/64 . 1/2 $\int$ dx – 1/64 . ½ $\int$ cos 8x dx + $\frac{(\text{sen}^3\, 2x)}{48}$ + 1/32 x - 1/128 sen 4x .

$\cancel{1/128\, x}$ – 1/128 . 1/8 $\int$ cos 8x . (8)dx + $\frac{(\text{sen}^3\, 2x)}{48}$ + $\cancel{1/32\, x}$ - 1/128 sen 4x .

5/128 x - 1/1024 sen 8x + $\frac{(\text{sen}^3\, 2x)}{48}$ - 1/128 sen 4x + c .

21. $\int$ **(1 + cos x)^3 . dx .**
$\int$ (1^3 + 3.1^2.cos x + 3.1.cos^2 x + cos^3 x) . dx .
$\int$ (1 + 3cos x + 3cos^2 x + cos^3 x) . dx .
$\int$ [1 + 3cos x + 3(½ + ½ cos 2x) + cos^2 x .cos x] . dx .
$\int$ [1 + 3cos x + 3(½ + ½ cos 2x) + (1 - sen^2 x) .cos x] . dx .
$\int$ [$\cancel{2/2}$ + $\cancel{3\cos}$ x + $\cancel{3/2}$ + 3/2 cos 2x + $\cancel{\cos}$ x - sen^2 x .cos x] . dx .
$\int$ [5/2 + 4cos x + 3/2 cos 2x - sen^2 x .cos x] . dx .
5/2 $\int$ dx + 4 $\int$ cos x dx + 3/2 . ½ $\int$ cos 2x .(2) dx - $\int$ (sen x)^2 .cos x . dx

$5/2\ x + 4\ \text{sen}\ x + \frac{3}{4}\ \text{sen}\ 2x - \frac{(\text{sen}\ x)^{2+1}}{2+1} + c$.

$\frac{5x}{2} + 4\ \text{sen}\ x + \frac{3\text{sen}\ 2x}{4} - \frac{(\text{sen}\ x)^3}{3} + c$.

22. $\int (\sqrt{\text{sen}\ 2\theta} - \cos 2\theta)^2\ d\theta$

$\int [(\sqrt{\text{sen}\ 2\theta})^2 - 2(\sqrt{\text{sen}\ 2\theta}) . \cos 2\theta + \cos^2 2\theta]\ d\theta$

$\int [\text{sen}\ 2\theta - 2(\text{sen}\ 2\theta)^{1/2} . \cos 2\theta + \cos^2 2\theta]\ d\theta$

$\int \{\text{sen}\ 2\theta - 2(\text{sen}\ 2\theta)^{1/2} . \cos 2\theta + [1/2 + \frac{1}{2} \cos 2(2\theta)]\}\ d\theta$

$\int \{\text{sen}\ 2\theta - 2(\text{sen}\ 2\theta)^{1/2} . \cos 2\theta + 1/2 + \frac{1}{2} \cos 4\theta\}\ d\theta$

$\int \text{sen}\ 2\theta\ .\ d\theta - 2.\frac{1}{2} \int (\text{sen}\ 2\theta)^{1/2}.\cos 2\theta.(2) + 1/2\int d\theta + \frac{1}{2}\ .1/4 \int \cos 4\theta.(4)\ d\theta$

$\frac{1}{2} \int \text{sen}\ 2\theta.(2).\ d\theta - \frac{(\text{sen}\ 2\theta)^{1/2+1}}{\frac{1}{2}+1} + \theta/2 + 1/8\ \text{sen}\ 4\theta\ d\theta$

$\frac{1}{2} (-\cos 2\theta) - \frac{(\text{sen}\ 2\theta)^{1/2+1}}{\frac{1}{2}+1} + \theta/2 + 1/8\ \text{sen}\ 4\theta\ d\theta$

$-\frac{1}{2} (\cos 2\theta) - \frac{(\text{sen}\ 2\theta)^{3/2}}{3/2} + \theta/2 + 1/8\ \text{sen}\ 4\theta\ d\theta$

$-\frac{1}{2} (\cos 2\theta) - 2\frac{(\text{sen}\ 2\theta)^{3/2}}{3} + \theta/2 + 1/8\ \text{sen}\ 4\theta + c$.

Ordenando:

$\theta/2 + 1/8\ \text{sen}\ 4\theta - 2\frac{(\text{sen}\ 2\theta)^{3/2}}{3} - \frac{1}{2} (\cos 2\theta) + c$.

23. $\int (\sqrt{\cos\theta} - 2\text{sen}\ \theta)^2\ d\theta$

$\int [(\sqrt{\cos\theta})^2 - 2\ (\sqrt{\cos\theta}\ .2\text{sen}\ \theta + (2\ \text{sen}\ \theta)^2]\ d\theta$

$\int [(\cos\theta) - 2.2\ (\cos\theta)^{1/2}.\text{sen}\ \theta + (4\ \text{sen}^2\ \theta)]\ d\theta$

$\int [\cos\theta - 4(\cos\theta)^{1/2}.\text{sen}\ \theta + 4(1/2 - 1/2 \cos 2\theta]\ d\theta$

$\int [(\cos\theta) - 4(\cos\theta)^{1/2}.\text{sen}\ \theta + 4/2 - 4/2 \cos 2\theta]\ d\theta$

$\int [\cos\theta - 4(\cos\theta)^{1/2}.\text{sen}\ \theta + 2 - 2 \cos 2\theta]\ d\theta$

$\int (\cos\theta)\ d\theta - 4\int (\cos\theta)^{1/2}.\text{sen}\ \theta\ d\theta + 2\int d\theta - 2.\frac{1}{2} \int \cos 2\theta.(2)\ .d\theta$

$\text{sen}\ \theta - \frac{4(\cos\theta)^{1/2+1}}{\frac{1}{2}+1} + 2\theta - \text{sen}\ 2\theta$

$\text{sen}\ \theta - \underline{4(\cos\theta)^{3/2}} + 2\theta - \text{sen}\ 2\theta + c$.

$$\text{sen}\,\theta - \frac{8(\cos\theta)^{3/2}}{3} + 2\theta - \text{sen}\,2\theta + c.$$

24. $\int (\text{sen}\,2x - \text{sen}\,3x)^2\,dx$

$\int (\text{sen}^2 2x - 2\,\text{sen}\,2x\,.\text{sen}\,3x + \text{sen}^2 3x)dx$

$\int \{(½ - ½\cos 2(2x) - 2[-½\cos(2+3)x + ½\cos(2-3)x] + [½ - ½\cos 2(3x)]dx$

$\int \{(½ - ½\cos 4x) - 2[-½\cos 5x + ½\cos(-x)] + [½ - ½\cos 6x]\}dx$

$\int \{½ - ½\cos 4x + \cos 5x - \cos(-x) + ½ - ½\cos 6x\}dx$

Por Trigonometría: $\cos(-x) = \cos(x)$.

$\int \{½ + ½ - ½\cos 4x + \cos 5x + \cos x - ½\cos 6x\}dx$

$\int \{1 - ½\cos 4x + \cos 5x + \cos x - ½\cos 6x\}dx$

$\int dx - ½.1/4 \int \cos 4x\,.(4)\,dx + 1/5 \int \cos 5x\,.(5)dx + \int \cos x.dx - ½\,.1/6 \int \cos 6x\,(6)dx$

$\int x\,dx - 1/8 \int \cos 4x\,.(4)\,dx + 1/5 \int \cos 5x\,.(5)dx + \int \cos x.dx - 1/12 \int \cos 6x\,.(6)dx$

$x - 1/8\,\text{sen}\,4x + 1/5\,\text{sen}\,5x + \text{sen}\,x - 1/12\,\text{sen}\,6x.$

$$x - \frac{\text{sen}\,4x}{8} + \frac{\text{sen}\,5x}{5} + \text{sen}\,x - \frac{\text{sen}\,6x}{12} + c.$$

25. $\int (\text{sen}\,x + \cos 2x)^2\,dx$

$\int (\text{sen}^2 x + 2\,\text{sen}\,x\,.\cos 2x + \cos^2 2x)\,dx$

Por Trigonometría: $\cos 2x = \cos^2 x - \text{sen}^2 x$; $\text{sen}^2 x = ½ - ½\cos 2x$; $\cos^2 x = ½ + ½\cos 2x$.

$\int [(½ - ½\cos 2x) + 2\,\text{sen}\,x\,(\cos^2 x - \text{sen}^2 x) + ½ + ½\cos 2(2x)]\,dx$

$\int [½ - ½\cos 2x + 2\cos^2 x\,.\text{sen}\,x - 2\text{sen}^2 x.\,\text{sen}x + ½ + ½\cos 4x]\,dx$

$\int [½ + ½ - ½\cos 2x + 2\cos^2 x\,.\text{sen}\,x - 2\text{sen}^2 x.\,\text{sen}x + ½\cos 4x]\,dx$

$\int [1 - ½\cos 2x + 2\cos^2 x\,.\text{sen}\,x - 2(1 - \cos^2 x).\text{sen}x + ½\cos 4x]\,dx$

$\int [1 - ½\cos 2x + 2\cos^2 x\,.\text{sen}\,x - 2\,\text{sen}\,x + 2\cos^2 x.\text{sen}x + ½\cos 4x]\,dx$

$\int [1 - ½\cos 2x + 4\cos^2 x\,.\text{sen}\,x - 2\,\text{sen}\,x + ½\cos 4x]\,dx.$

$\int dx - ½\,.\,½ \int \cos 2x\,.(2)dx + (-)4 \int (\cos x)^2\,.(-)\text{sen}\,x\,dx - 2\int \text{sen}\,x\,.dx + ½.1/4 \int \cos 4x.(4)\,dx$

$\int dx - 1/4\int \cos 2x\,.(2)dx - 4\int (\cos x)^2.(-)\text{sen}\,x\,dx - 2\int \text{sen}\,x\,.dx + 1/8\int \cos 4x.(4)\,dx$

$$x - \frac{\text{sen}\,2x}{4} - \frac{4(\cos x)^3}{3} - 2(-\cos x) + \frac{\text{sen}\,4x}{8} + c.$$

$$x - \frac{\text{sen } 2x}{4} - \frac{4(\cos x)^3}{3} + 2\cos x + \frac{\text{sen } 4x}{8} + c\ .$$

26. $\int (\cos x + 2\cos 2x)^2\, dx$

$\int [(\cos x)^2 + 2(\cos x).(2\cos 2x) + (2\cos 2x)^2\, dx$

$\int (\cos^2 x + 4\cos x\ .\cos 2x + 4\cos^2 2x)\, dx$

$\int (\frac{1}{2} + \frac{1}{2}\cos 2x + 4\cos x\ .(\cos^2 x - \text{sen}^2 x) + 4(\frac{1}{2} + \frac{1}{2}\cos 2(2x)\, dx$

$\int \frac{1}{2} + \frac{1}{2}\cos 2x + 4\cos^2 x.\ \cos x - 4\text{sen}^2 x.\cos x + 2 + 2\cos 4x]\, dx$

$\int \frac{1}{2} + 2 + \frac{1}{2}\cos 2x + 4\cos^2 x.\ \cos x - 4\text{sen}^2 x.\cos x + 2\cos 4x]\, dx$

$\int 5/2 + \frac{1}{2}\cos 2x + 4\cos^2 x.\ \cos x - 4\text{sen}^2 x.\cos x + 2\cos 4x]\, dx$

$\int [5/2 + \frac{1}{2}\cos 2x + 4(1 - \text{sen}^2 x)\ .\ \cos x - 4\text{sen}^2 x.\cos x + 2\cos 4x]\, dx$

$\int [5/2 + \frac{1}{2}\cos 2x + 4\cos x - 4\text{sen}^2 x\ .\cos x - 4\text{sen}^2 x.\cos x + 2\cos 4x]\, dx$

$\int [5/2 + \frac{1}{2}\cos 2x + 4\cos x - 8\text{sen}^2 x\ .\cos x + 2\cos 4x]\, dx$

$5/2\int dx + \frac{1}{2}.\frac{1}{2}\int \cos 2x.(2)dx + 4(-)\int \cos x.(-)dx - 8\int (\text{sen } x)^2\ .\cos x.dx$
$+ 2.1/4 \int \cos 4x.(4)dx$

$5/2\int dx + 1/4\int \cos 2x.(2)dx - 4\int \cos x.(-)dx - 8\int (\text{sen } x)^2.\cos x.dx + \frac{1}{2}\int \cos 4x.$
$(4)dx$

$$\frac{5x}{2} + \frac{\text{sen } 2x}{4} - 4\text{sen } x - \frac{8(\text{sen } x)^{2+1}}{2+1} + \frac{\text{sen } 4x}{2} + c\ .$$

$$\frac{5x}{2} + \frac{\text{sen } 2x}{4} - 4\text{sen } x - \frac{8(\text{sen } x)^3}{3} + \frac{\text{sen } 4x}{2} + c\ .$$

Problemas. Páginas 268 - 269

Cuando ocurre $\sqrt{a^2 - u^2}$; hágase u = a sen z
Cuando ocurre $\sqrt{a^2 + u^2}$; hágase u = a tg z
Cuando ocurre $\sqrt{u^2 - a^2}$; hágase u = a sec z
En efecto:

$\sqrt{a^2 - a^2 \operatorname{sen}^2 z} = a\sqrt{1 - \operatorname{sen}^2 z} = a \cos z$ (1)

$\sqrt{a^2 + a^2 \operatorname{tg}^2 z} = a\sqrt{1 + \operatorname{tg}^2 z} = a \sec z$ (2)

$\sqrt{a^2 \sec^2 z - a^2} = a\sqrt{\sec^2 z - 1} = a \operatorname{tg} z$ (3)

1.- $\displaystyle\int \frac{dx}{(x^2+2)^{3/2}}$.

$\left.\begin{array}{l} u = x \\ a = \sqrt{2} \\ a^2 = 2 \end{array}\right\}$ Como: $(x^2 + 2)^{3/2} = \{\sqrt{(x^2 + 2)}\}^3$ es similar a $\sqrt{a^2 + u^2}$ □ hágase u = a tg z

$u = a \operatorname{tg} z$

$du = a\sec^2 z\, dz\, dz.$

$dx = du$

Sustituyendo,haciendo operaciones y utilizando (2) resulta:

$$\int \frac{dx}{(x^2+2)^{3/2}} = \int \frac{du}{\{\sqrt{(u^2+a^2)}\}^3} = \int \frac{a\sec^2 z\, dz\, dz}{\{a \sec z\}^3} = \int \frac{\mathbf{a}\sec^2 z\, dz}{a^3 \sec^3 z}$$

=

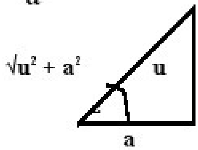
$$\int \frac{\cancel{\sec^2} z\, dz}{a^2\, \cancel{\sec^3} z} = \frac{1}{a^2}\int \frac{dz}{\sec z} = \frac{1}{a^2}\int \cos z\, dz = \frac{\text{sen } z}{a^2} +$$

$u = a \operatorname{tg} z$.

$a \operatorname{tg} z = u$

$\operatorname{tg} z = \dfrac{u}{a}$.

$\text{sen } z = \dfrac{u}{\sqrt{u^2+a^2}}$.



$$\frac{\text{sen } z}{a^2} = \frac{\dfrac{u}{\sqrt{u^2+a^2}}}{a^2} = \frac{u}{a^2\sqrt{u^2+a^2}} = \frac{x}{2\sqrt{x^2+4}} + c \;. =$$

2.- $\displaystyle\int \frac{\mathbf{x^2\, dx}}{\sqrt{(\mathbf{x^2-6})}} = x\sqrt{(x^2-6)} + 3\ln\left(x + \sqrt{(x^2-6)}\right) + c$.

$u = x$

$a = \sqrt{6}$

$a^2 = 6$

Como: $\sqrt{(x^2-6)} =$ es similar a $\sqrt{u^2-a^2}$

□ hágase $u = a \sec z$

$u = a \sec z$

$du = a \sec z \,.\operatorname{tg} z\, dz$.

$dx = du$

Sustituyendo,haciendo operaciones y utilizando (3) resulta:

$$\int \frac{x^2\,dx}{\sqrt{(x^2-6)}} = \int \frac{u^2\, du}{\sqrt{u^2-a^2}} = \int \frac{(a\sec z)^2 . a \sec z\, \cancel{\operatorname{tg} z}\, dz}{a\, \cancel{\operatorname{tg} z}}\, dz =$$

$\int a^2 \sec^2 z \,.\, \sec z\, dz = a^2 \int \sec^3 z\, dz =$

Se integra por partes:

$\mathbf{\int u\, dv = uv - \int v\, du}$

$\int \sec^3 z\, dz = \int \sec z \,.\, \sec^2 z\, dz =$

$u = \sec z$ $\qquad dv = \sec^2 z\, dz$

$du = \sec z \,.\, \operatorname{tg} z\, dz$ $\qquad v = \operatorname{tg} z$

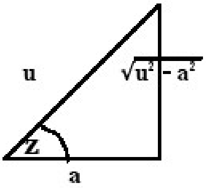
$$\int \sec^3 z\, dz = \sec z \,.\, tg\, z - \int tg\, z \,.\sec z \,.\, tg\, z\, dz =$$
$$\int \sec^3 z\, dz = \sec z \,.\, tg\, z - \int tg^2\, z \,.\sec z \,.dz =$$
$$\int \sec^3 z\, dz = \sec z \,.\, tg\, z - \int (\sec^2 z - 1) \,.\sec z \,.dz =$$
$$\int \sec^3 z\, dz = \sec z \,.\, tg\, z - [\int (\sec^3 z - \sec z) \,.dz] =$$
$$\int \sec^3 z\, dz = \sec z \,.\, tg\, z - \int \sec^3 z\, dz - \int \sec z \,.dz =$$
$$\int \sec^3 z\, dz + \int \sec^3 z\, dz = \sec z \,.\, tg\, z - \int \sec z \,.dz =$$
$$2 \int \sec^3 z\, dz = \sec z \,.\, tg\, z - \ln (\sec z + tg\, z) =$$
$$\int \sec^3 z\, dz = \frac{\sec z \,.\, tg\, z - \ln (\sec z + tg\, z)}{2} =$$

Pero: $a^2 \int \sec^3 z\, dz = a^2 \left\{ \frac{\sec z \,.\, tg\, z - \ln (\sec z + tg\, z)}{2} = \right\}$

$$\frac{a^2 \sec z \,.\, tg\, z}{2} - \frac{a^2 \ln (\sec z + tg\, z)}{2} =$$

$u = a \sec z$

$\sec z = \frac{u}{a}$; $tg\, z = \frac{\sqrt{u^2 - a^2}}{a}$



$$\int \frac{x^2\, dx}{\sqrt{(x^2 - 6)}} = \frac{a^2 \sec z \,.\, tg\, z}{2} - \frac{a^2 \ln (\sec z + tg\, z)}{2} =$$

$$= \frac{\frac{a^2\, u \,.\, \sqrt{u^2 - a^2}}{a.a}}{\frac{2}{1}} - \frac{a^2 \ln \left(\frac{u}{a} + \frac{\sqrt{u^2 - a^2}}{a} = \right)}{\frac{2}{1}} =$$

$$\frac{\cancel{a^2}\, u \,.\sqrt{u^2 - a^2}}{2\cancel{a^2}} - \frac{a^2}{2} \ln \left\{ \frac{u + \sqrt{u^2 - a^2}}{a} \right\}$$

$$\frac{u.\sqrt{u^2 - a^2}}{2} - \frac{a^2}{2} \ln \left\{ \frac{u + \sqrt{u^2 - a^2}}{a} \right\}$$

Sustituyendo valores:

$$\frac{x\,.\,\sqrt{x^2-6}}{2}-\frac{6}{2}\left\{\ln\,\frac{x+\sqrt{x^2-6}}{\sqrt{6}}\right\}$$

Aplicando logaritmos naturales:

$$\frac{x\,.\,\sqrt{x^2-6}}{2}-3\ln\left[x+\sqrt{x^2-6}\right]-3\,\ln\sqrt{6}+c\,.\ \text{Pero: } 3\,\ln\sqrt{6}=c$$

$$\frac{x\,.\,\sqrt{x^2-6}}{2}-3\ln\left[x+\sqrt{x^2-6}\right]+c\,.$$

3.- $\displaystyle\int\frac{dx}{(5-x^2)^{3/2}}\,.$

$\left.\begin{array}{l} u=x \\ a=\sqrt{5} \\ a^2=5 \end{array}\right\}$ Como: $(5-x^2)^{3/2}=\{\sqrt{(5-x^2}\}^3$ es similar a $\sqrt{a^2-u^2}$ □ hágase $u=a\,\text{sen}\,z$

$u=a\,\text{sen}\,z$

$du=a\cos z\,dz.$

$du=dx$

Sustituyendo, haciendo operaciones y utilizando (1) resulta:

$$\int\frac{dx}{(5-x^2)^{3/2}}=\int\frac{du}{\{\sqrt{a^2-u^2}\}^3}=\int\frac{a\cos z\,dz}{\{a\cos z\}^3}=\int\frac{a\cos z\,dz}{a^3\cos^3 z}=$$

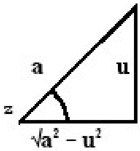
$$\int\frac{dz}{a^2\cos^2 z}=\frac{1}{a^2}\int\sec^2 z\,dz=\frac{1}{a^2}\,\text{tg}\,z=\frac{\text{tg}\,z}{a^2}\,.$$

$u=a\,\text{sen}\,z\,.$

$a\,\text{sen}\,z=u$

$\text{sen}\,z=\dfrac{u}{a}\,.$

$\text{tg}\,z=\dfrac{u}{\sqrt{a^2-u^2}}\,.$



$$\frac{\text{tg}\,z}{a^2}=\frac{\dfrac{u}{\sqrt{a^2-u^2}}}{a^2}=\frac{u}{a^2\sqrt{a^2-u^2}}=\frac{x}{5\sqrt{5-x^2}}+c\,.$$

4.- $\displaystyle\int \frac{t^2\,dt}{\sqrt{(4 - t^2)}} =$

$u = t$; $a = 2$; $a^2 = 4$ } Como: $\sqrt{(4 - t^2)}$ = es similar a $\sqrt{a^2 - u^2}$

□ hágase $u = a \operatorname{sen} z$

$u = a \operatorname{sen} z$

$du = a \cos z\, dz$.

$du = dt$

Sustituyendo, haciendo operaciones y utilizando (1) resulta:

$$\int \frac{t^2\,dx}{\sqrt{(4 - t^2)}} = \int \frac{u^2\,du}{\sqrt{a^2 - u^2}} = \int \frac{(a \operatorname{sen} z)^2 \cdot \cancel{a \cos z}\, dz}{\cancel{a \cos z}} =$$

Aplicando la formula: $\int \operatorname{sen}^2 u\, du = \frac{1}{2} u - \frac{1}{4} \operatorname{sen} 2u + c$

$$\int a^2 \operatorname{sen}^2 z\, .dz = a^2 (\tfrac{1}{2} - \tfrac{1}{4} \operatorname{sen} 2z) =$$

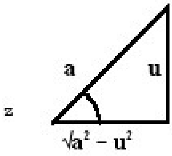
$u = a \operatorname{sen} z$.

$a \operatorname{sen} z = u$

$\operatorname{sen} z = \dfrac{u}{a}$.

$\cos z = \dfrac{\sqrt{a^2 - u^2}}{a}$.

$z = \operatorname{arc\,sen} u/a$



$\operatorname{sen} 2z = 2 \operatorname{sen} z \cdot \cos z$

$a^2 (\frac{1}{2} z - \frac{1}{4} \operatorname{sen} 2z) = a^2 [\frac{1}{2} z - \frac{1}{4} (2 \operatorname{sen} z \cdot \cos z)] =$

$a^2 [\frac{1}{2} z - \frac{1}{2} \operatorname{sen} z \cdot \cos z)] = \dfrac{a^2 z}{2} - \dfrac{a^2 \operatorname{sen} z \cdot \cos z)]}{2} =$

$\dfrac{4z}{2} - \dfrac{4}{2} \dfrac{u}{a} \cdot \dfrac{\sqrt{a^2 - u^2}}{a} = 2z - 2u \cdot \dfrac{\sqrt{a^2 - u^2}}{a^2} =$

Pero: $z = \operatorname{arc\,sen} u/a$; $u = t$; $a^2 = 4$

$2 \operatorname{arc\,sen} u/a - \dfrac{2t \cdot \sqrt{4 - t^2}}{4} = 2 \operatorname{arc\,sen} t/2 - \dfrac{t}{2} \cdot \sqrt{4 - t^2} + c$.

Problemas. Páginas 272, 273, 274 . "Integración por Partes"

Para esta clase de problemas se aplica la siguiente fórmula:
$\int u \, . \, dv = u.v - \int v \, . \, du$.

1.- $\int x \operatorname{sen} x \, dx$

$u = x \qquad dv = \operatorname{sen} x \, dx$

Para obtener "v" , se integra dv = senx ,en ambos miembros.
De igual modo se efectua en todos los problemas.

$u = x \qquad \int dv = \int \operatorname{sen} x \, dx$

$du = dx \qquad v = - \cos x$

$x(- \cos x) - \int - \cos x \, . \, dx = - x \, . \cos x + \int \cos x \, . \, dx =$
$- x \, .\cos x + \operatorname{sen} x = \operatorname{sen} x - x \cos x + c$.

2.- $\int \ln x \, dx = x (\ln x - 1) + c$.

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} \, dx \qquad v = x$

$\ln x \, . \, x - \int x \, . \frac{1}{x} dx = \ln x \, . \, x - \int dx = x \ln x - x = x(\ln x - 1) + c$.

3.- $\int x \operatorname{sen} \frac{x}{2} \, dx = 4 \operatorname{sen} \frac{x}{2} - 2x \cos \frac{x}{2} \, d + c$.

$u = x \qquad \int dv = 2\int \operatorname{sen} \frac{x}{2} \, .½ \, dx$

$du = dx \qquad v = - 2 \cos \frac{x}{2}$

$x \left(- 2 \cos \frac{x}{2}\right) - \int 2 \, .\cos \frac{x}{2} \, . \, dx = - 2x \cos \frac{x}{2} - 2 \, .2 \int \cos \frac{x}{2} \, .(½) \, dx$

$- 2x \cos \frac{x}{2} - 4 \int \cos \frac{x}{2} \, .(½) \, dx = - 2x \cos \frac{x}{2} - 4\left(- \operatorname{sen} \frac{x}{2} \right).$

$- 2x \cos \frac{x}{2} + 4 \operatorname{sen} \frac{x}{2} = 4 \operatorname{sen} \frac{x}{2} - 2x \cos \frac{x}{2} + c$.

4.- $\int x \cos nx\, dx = \frac{\cos nx}{n^2} + x \operatorname{sen} \frac{nx}{n} + c$ **.**

$u = x$ $\qquad$ $dv = \cos nx \, . \, dx$

$du = dx$ $\qquad$ $\int dv = 1/n \int \cos nx \,.(n)\, dx$

$du = dx$ $\qquad$ $v = \operatorname{sen} \frac{nx}{n}$

$x \, . \operatorname{sen} \frac{nx}{n} - \int \frac{\operatorname{sen} nx}{n} \, . \, dx = x \,.\operatorname{sen} \frac{nx}{n} - \left(\frac{1}{n}\right)\left(\frac{1}{n}\right) \int \operatorname{sen} nx \,.(n)dx =$

$\frac{x \,.\operatorname{sen} nx}{n} - \frac{1}{n^2}\left(- \cos nx\right) = \frac{x \,.\operatorname{sen} nx}{n} + \frac{\cos nx}{n^2} + c \, . =$

5.- $\int u \sec^2 u \, du = u \operatorname{tg} u + \ln \cos u + c$ **.**

$u = u$ $\qquad$ $dv = \sec^2 u \, du$

$\int dv = \int \sec^2 u \, du$

$du = du$ $\qquad$ $v = \operatorname{tg} u$

$u \,.\operatorname{tg} u - \int \operatorname{tg} u \, du = u \,.\operatorname{tg} u - (- \ln \cos u) = u \,.\operatorname{tg} u + \ln \cos u + c$.

6.- $\int v \operatorname{sen}^2 3v \, dv = ¼ v^2 - 1/12 \, v \operatorname{sen} 6v - 1/72 \cos 6v \, u + c$ **.**

$u = v$ $\qquad$ $dv = \operatorname{sen}^2 3v \, dv$

$du = dv$ $\qquad$ $\int dv = \int \operatorname{sen}^2 3v \, dv$

Se aplica : $\int \operatorname{sen}^2 u \, du = ½ u - ¼ \operatorname{sen} 2u$; donde $u = 3v$

$v = 1/3 \int \operatorname{sen}^2 3v \,.(3)\, dv$

$v = 1/3 \, [½ \, 3v - ¼ \operatorname{sen} 2(3v)]$

$v = 1/6 \, 3v - 1/12 \operatorname{sen} 6v$

$v = ½ v - 1/12 \operatorname{sen} 6v$

$v\, (½ v - 1/12 \operatorname{sen} 6v) - \int (½ v - 1/12 \operatorname{sen} 6v)\, dv =$

$½ v^2 (- \underline{v \,.\operatorname{sen} 6v}) - ½ \int v \, dv + 1/12 \,.1/6 \int \operatorname{sen} 6v \,.(6)\, dv =$

7.- $\int y^2 \text{ sen } ny\, dy = 2\cos ny + \frac{2y \text{ sen } ny}{n^2} - \frac{y^2\cos ny}{n} + c$.

$u = y^2$ $\qquad$ $dv = \text{sen } ny\, dy$

$du = 2y\, .dy$ $\qquad$ $\int dv = \int \text{sen } ny\, dy$

$v = (1/n)\int \text{sen } ny\, .(n)\, dy$

$v = \frac{(-\cos ny)}{n}$

$\frac{y^2(-\cos ny)}{n} - \int \frac{(-\cos ny)}{n} 2ydy =$

$-\frac{y^2\cos ny}{n} + \left(\frac{2}{n}\right)\int \cos ny\, .y\, dy = -\frac{y^2\cos ny}{n} + \left(\frac{2}{n}\right)\int y \cos ny\, .\, dy$

Integrando por partes : $\int$ **y** cos ny .dy .

$u = y$ $\qquad$ $dv = \cos ny\, dy$

$du = dy$ $\qquad$ $\int dv = \int \cos ny\, dy$

$v = (1/n)\int \cos ny\, .(n)\, dy$

$v = \frac{(\text{sen } ny)}{n}$

$\int y \cos ny.dy = \frac{y\, (\text{sen } ny)}{n} - \int \frac{(\text{sen } ny)}{n}.dy =$

$\int y \cos ny.dy = \frac{y\, (\text{sen } ny)}{n} - \frac{1}{n}.\frac{1}{n}.\int (\text{sen } ny)\, .(n)dy$

$\int y \cos ny.dy = \frac{y\, (\text{sen } ny)}{n} - \frac{1}{n^2}.\, (-\cos ny)\, .(n)dy$

$\int y \cos ny.dy = \frac{y\, (\text{sen } ny)}{n} + \frac{(\cos ny)}{n^2}$

Enlazando y sustituyendo $\int$ y cos ny.dy , en la integral original:

$$\int \mathbf{y^2\ sen\ ny\ dy} = -\frac{y^2 \cos ny}{n} + \left(\frac{2}{n}\right)\int \mathbf{y \cos ny\ .\ dy}$$

$$\int \mathbf{y^2\ sen\ ny\ dy} = -\frac{y^2 \cos ny}{n} + \left(\frac{2}{n}\right)\left(\frac{y\,(\text{sen } ny)}{n} + \frac{(\cos ny)}{n^2}\right) =$$

$$\int \mathbf{y^2\ sen\ ny\ dy} = -\frac{y^2 \cos ny}{n} + \frac{2\ y\ \text{sen } ny}{n^2} + \frac{2 \cos ny}{n^3}\ \text{.Ordenando:}$$

$$\int \mathbf{y^2\ sen\ ny\ dy} = \frac{2 \cos ny}{n^3} + \frac{2\ y\ \text{sen } ny}{n^2} - \frac{y^2 \cos ny}{n} + c\ .$$

8.- $$\int \mathbf{x\ a^x\ dx} = \mathbf{a^x}\left(\frac{\mathbf{x}}{\mathbf{\ln a}} - \frac{\mathbf{1}}{\mathbf{\ln^2 a}}\right) + \mathbf{c}\ .$$

$u = x$ $\qquad\qquad$ $dv = a^x\ dx$

$du = dx$ $\qquad\qquad$ $\int dv = \int a^x\ dx$

$v = \dfrac{a^x}{\ln a}$

$$\frac{x\ .\ a^x}{\ln a} - \int \frac{a^x}{\ln a\ \ n}.\ dx = \frac{x\ .\ a^x}{\ln a} - \frac{1}{\ln a}\int a^x\ .dx = \frac{x\ .\ a^x}{\ln a} - \frac{1}{\ln a}\left(\frac{a^x}{\ln a}\right)\ .$$

$$\frac{x\ .\ a^x}{\ln a} - \frac{a^x}{\ln^2 a} = a^x\left(\frac{x}{\ln a} - \frac{1}{\ln^2 a}\right) + c\ .$$

9.- $$\int \mathbf{x^n\ \ln x\ dx} = \frac{\mathbf{x^{n+1}}}{\mathbf{n+1}}\left(\mathbf{\ln x} - \frac{\mathbf{1}}{\mathbf{n+1}}\right) + \mathbf{c}\ .$$

$\int x^n\ \ln x\ dx =$

$u = \ln x$ $\qquad\qquad$ $dv = x^n\ dx$

$du = \dfrac{1}{x}dx$ $\qquad\qquad$ $\int dv = \int x^n\ dx$

$v = \dfrac{x^{n+1}}{n+1}$

$$\ln x\ .\ \frac{x^{n+1}}{} - \int\left(\frac{x^{n+1}}{}\right).\left(\frac{1}{}\right) dx = \ln x\ .\ \frac{x^{n+1}}{} - \frac{1}{}\int \frac{x^{n+1}}{}\ dx$$

n+1   n+1   x   n+1   n+1   x

$$\frac{\ln x \cdot x^{n+1}}{n+1} - \frac{1}{n+1}\int\left(x^{n+1}.x^{-1}\right)dx = \ln x \cdot \frac{x^{n+1}}{n+1} - \frac{1}{n+1}\int x^{n+1-1}\,dx =$$

$$\ln x \cdot \frac{x^{n+1}}{n+1} - \frac{1}{n+1}\int x^{n}\,dx = \ln x \cdot \frac{x^{n+1}}{n+1} - \frac{1}{n+1}.\left(\frac{x^{n+1}}{n+1}\right) =$$

$$\frac{x^{n+1}}{n+1}\left(\ln x - \frac{1}{n+1}\right) + c\,.$$

10.- $\int$ **arc sen x dx = x arc sen x +** $\sqrt{1 - x^2}$ **+ c .**

u = arc sen x   dv = dx

$du = \frac{1}{\sqrt{1 - x^2}}\,dx$ $\int dv = \int dx$

v = x

$$\text{arc sen } x \cdot x - \int x \cdot \frac{1}{\sqrt{1 - x^2}}\,dx = x\ \text{arc sen } x - \int x\,(1 - x^2)^{-1/2}\,dx =$$

Ordenando: $\int x\,(1 - x^2)^{-1/2}\,dx$ y completando el diferencial.

$$x\ \text{arc sen } x - (-\tfrac{1}{2})\int (1- x^2)^{-1/2}.(-2)x\,dx = x\ \text{arc sen } x + \frac{\tfrac{1}{2}(1- x^2)^{-1/2+1}}{-1/2+1}$$

$$x\ \text{arc sen } x + \frac{(1- x^2)^{1/2}}{2(1/2)} = x\ \text{arc sen } x + (1- x^2)^{1/2} =$$

$$x\ \text{arc sen } x + \sqrt{1- x^2} + c\,.$$

11.- $\int$ **arc tg x dx = x arc tg x - ½ ln (1 +** x^2**) + c .**

u = arc tg x   dv = dx

$du = \frac{1}{1 + x^2}\,dx$ $\int dv = \int dx$

v = x

$$\text{arc tg } x \cdot x - \int\left(x \cdot \frac{1}{1 + x^2}\right)dx\,.$$

$\text{arc tg } x \cdot x - \int \frac{x}{\quad}\,dx$. Completando el diferencial.

$$(1+x^2)$$

$v = 1 + x^2$ Falta (2) para completar el diferencial.Se aplica:

$dv = 2x\,dx$ $\int \frac{dv}{v} = \ln v + c$

$\text{arc tg } x \,.\, x - (½)\int \frac{(2)x}{(1+x^2)}\, dx$. Completando el diferencial y ordenando.

$x \text{ arc tg } x - ½ \ln (1 + x^2) = x \text{ arc sen } x - ½ \ln (1 + x^2) + c$.

12.- $\int$ **arc cot y dy = y arc cot y + ½ ln (1 + y²) + c .**

$u = \text{arc cot } y$ | $dv = dy$

$du = \left(- \frac{1}{1+y^2}\right)dx$ | $\int dv = \int dy$

$v = y$

$\text{arc cot } y \,.\, y - \int y \left(- \frac{1}{1+x^2}\right) dx$.

$y \,.\text{arc cot } y + \int \frac{y}{(1+y^2)} \,.\, dy$. Completando el diferencial.

$v = 1 + y^2$ Falta (2) para completar el diferencial.Se aplica:

$dv = 2y\,dy$ $\int \frac{dv}{v} = \ln v + c$

$y \,.\text{arc cot } y + \int \frac{y}{(1+y^2)} \,.\, dy$. Completando el diferencial

$y \text{ arc cot } y + ½ \ln (1 + y^2) + c$.

13.- $\int$ **arc cos 2x ax . dx = x arc cos 2x - ½** $\mathbf{\sqrt{1 - 4x^2} + c}$ **.**

$u = \text{arc cos } 2x$ | $dv = dx$

$du = \left(- \frac{2}{\sqrt{1-(2x)^2}}\right).dx$ | $\int dv = \int dx$

$v = x$

$du = \left(- \frac{2}{\sqrt{1-4x^2}}\right).dx$

$$\text{arc cos } 2x \cdot x - \int x\left(-\frac{2}{\sqrt{1-4x^2}}\right) dx .$$

$$x \text{ arc cos } 2x + \int \frac{2x}{\sqrt{1-4x^2}} dx .$$

$x \text{ arc cos } 2x + \int (1 - 4x^2)^{-1/2} \cdot 2x\, dx$ .Completando el diferencial.

$\left.\begin{array}{l} v = 1 - 4x^2 \\ dv = -8x\, dx \end{array}\right\}$ Falta (- 4) para completar el diferencial.Se aplica: $\int v^n\, dv = \dfrac{v^{n+1}}{n+1} + c$

$x \text{ arc cos } 2x + (-\tfrac{1}{4}) \int (1 - 4x^2)^{-1/2} \cdot (-4)\, 2x\, dx$.

$x \text{ arc cos } 2x + (-\tfrac{1}{4}) \dfrac{(1 - 4x^2)^{-1/2+1}}{-1/2+1} = x \text{ arc cos } 2x - \tfrac{1}{4} \cdot \dfrac{(1 - 4x^2)^{1/2}}{1/2} =$

$x \text{ arc cos } 2x - (\tfrac{1}{4})(2)(1 - 4x^2)^{1/2} = x \text{ arc cos } 2x - \tfrac{1}{2} (1 - 4x^2)^{1/2}$

$\mathbf{x \text{ arc cos } 2x - \tfrac{1}{2} \sqrt{1 - 4x^2} + c .}$

14.- $\int \textbf{arc sec } \mathbf{y \cdot dy = y} \textbf{ arc sec } \mathbf{y - \ln (y + \sqrt{y^2 - 1} + c .}$

$u = \text{arc sec } y \qquad\qquad dv = dy$

$du = \left(\dfrac{1}{y\sqrt{y^2 - 1}}\right) dx \qquad \int dv = \int dy$

$\qquad\qquad\qquad\qquad v = y$

$$\text{arc sec } y \cdot y - \int y\left(\frac{1}{y\sqrt{y^2-1}}\right) dy .$$

$$\text{arc sec } y \cdot y - \int\left(\frac{y}{y\sqrt{y^2-1}}\right) dy .$$

$$\text{arc sec } y \cdot y - \int\left(\frac{\cancel{y}}{\cancel{y}\sqrt{y^2-1}}\right) dy .$$

$$y \text{ arc sec } y - \int \frac{dy}{\sqrt{y^2-1}} .$$

Se aplica : $\int \dfrac{dv}{\sqrt{v^2 - 1}} = \ln [v + \sqrt{v^2 - 1}\ ]$

$y \text{ arc sec } y - \ln [y + \sqrt{y^2 - 1}\,] + c$.

15.- $\int \text{arc csc } \frac{t}{2} . dt = t \text{ arc csc } \frac{t}{2} + 2 \ln (t + \sqrt{t^2 - 4}\,) + c$ **.**

$u = \text{arc csc} \frac{t}{2}$. $\qquad dv = dt$

$du = \left(\frac{-\frac{1}{2}}{\frac{1}{2} t \sqrt{(\frac{1}{2} t)^2 - 1}} \right) dt$. $\qquad \int dv = \int dt$

$v = t$

$\text{arc csc} \frac{t}{2} . t - \int t \left(\frac{-\frac{1}{2}}{\frac{1}{2} t \sqrt{(\frac{1}{2} t)^2 - 1}} \right) dt$. _

$\text{arc csc} \frac{t}{2} . t - \int \cancel{t} \left(\frac{\cancel{-\frac{1}{2}}}{\cancel{\frac{1}{2} t} \sqrt{(\frac{1}{2} t)^2 - 1}} \right) dt$. _

$\text{arc csc} \frac{t}{2} . t + \int \frac{dt}{\sqrt{\frac{1}{4} t^2 - 1}} =$ _

$\left. \begin{array}{l} v = \frac{1}{2} t \\ dv = \frac{1}{2} \end{array} \right\}$ Falta (½) para completar el diferencial.Se aplica:

$\int \frac{dv}{\sqrt{v^2 - a^2}} = \ln [v + \sqrt{(\frac{1}{2} t)^2 - 1} + c$

$t \text{ arc csc} \frac{t}{2} + 2 \int \frac{(\frac{1}{2})\, dt}{\sqrt{\frac{t^2 - 4}{4}}} = t \text{ arc csc} \frac{t}{2} + 2 \ln [v + \sqrt{(\frac{1}{2} t)^2 - 1}\,] + c$

16.- $\int x \text{ arc csc } x . dx = \frac{x^2 + 1}{2} \text{ arc tg } x - \frac{x}{2} + c$ **.**

$u = \text{arc csc } x$. $\qquad dv = dx$.

$du = \left(\frac{-1}{x \sqrt{x^2 - 1}} \right) dx$. $\qquad \int dv = \int dx$

$v = x$

$\text{arc csc } x \;. x - \int x \left(\frac{-1}{x \sqrt{x^2 - 1}} \right) dt$. _

$$\operatorname{arc\,csc}\frac{t}{3}\,.\,t-\int \cancel{t}\left(\frac{\cancel{-\tfrac{1}{2}}}{\cancel{\tfrac{1}{2}}\,t\sqrt{(\tfrac{1}{2}t)^2-1}}\right)dt\,.\,\_$$

$$\operatorname{arc\,csc}\frac{t}{2}\,.\,t+\int\frac{dt}{\sqrt{\tfrac{1}{4}t^2-1}}=\_$$

$$\left.\begin{array}{l} v=\tfrac{1}{2}t\\ dv=\tfrac{1}{2}\end{array}\right\}$$ Falta (½) para completar el diferencial.Se aplica: $\int\frac{dv}{\sqrt{v^2-a^2}}=\ln\left[v+\sqrt{(\tfrac{1}{2}t)^2-1}+c\right.$

18. $\int x^2e^{-x}\,dx=-e^{-x}\left(2+2x+x^2\right)+c.$

$u=x^2$ $\qquad\int dv=\int e^{-x}(-)\,dx$

$du=2x.dx$ $\qquad v=-e^{-x}$

$(x^2)(-e^{-x})-\int(-e^{-x})(2x)\,dx=-x^2.e^{-x}+2\int x.e^{-x}\,dx.$ (1ra solución).

$u=x$ $\qquad\int dv=\int e^{-x}.(-)\,dx$

$du=dx$ $\qquad v=-e^{-x}.$

$(x)(-e^{-x})-\int -e^{-x}\,dx=-x\,e^{-x}+(-)\int(e^{-x})(-)\,dx=$

$-xe^{-x}-e^{-x}$ (2da solución). Tomando contacto con la 1ra solución.

$-x^2.e^{-x}+2\left[-xe^{-x}-e^{-x}\right]=-x^2.e^{-x}-2xe^{-x}-2e^{-x}.$

$-e^{-x}\left[x^2+2x+2\right]+c.$

19. $\int e^{\theta}\cos\theta\,d\theta=\frac{e^{\theta}}{2}(\operatorname{sen}\theta+\cos\theta)+c.$

$u=e^{\theta}$ $\qquad\int dv=\int\cos\theta$

$du=e^{\theta}.d\theta$ $\qquad v=\operatorname{sen}\theta$

$\int e^{\theta}\cos\theta\,d\theta=e^{\theta}\operatorname{sen}\theta-\int e^{\theta}\operatorname{sen}\theta\,d\theta$

$$t\operatorname{arc\,csc}\frac{t}{2}+2\int\frac{(\tfrac{1}{2})\,dt}{\sqrt{\frac{t^2-4}{4}}}=t\operatorname{arc\,csc}\frac{t}{2}+2\ln\left[v+\sqrt{(\tfrac{1}{2}t)^2-1}\right]+c$$

$$u = e^{\theta} \qquad \int dv = \operatorname{sen}\Theta\, d\Theta$$

$$du = e^{\theta}\, d\Theta \qquad v = -\cos\Theta$$

$$\int e^{\theta}\cos\Theta . d\Theta = e^{\theta}\operatorname{sen}\Theta - \left[-e^{\theta}\cos\Theta - \int -e^{\theta}\cos\Theta\, d\Theta\right].$$

$$\int e^{\theta}\cos\Theta . d\Theta = e^{\theta}\operatorname{sen}\Theta - \left[-e^{\theta}\cos\Theta + \int e^{\theta}\cos\Theta\, d\Theta\right].$$

$$\int e^{\theta}\cos\Theta . d\Theta = e^{\theta}\operatorname{sen}\Theta + e^{\theta}\cos\Theta - \int e^{\theta}\cos\Theta\, d\Theta.$$

$$\int e^{\theta}\cos\Theta . d\Theta + \int e^{\theta}\cos\Theta . d\Theta = e^{\theta}(\operatorname{sen}\Theta + \cos\Theta).$$

$$2\int e^{\theta}\cos\Theta . d\Theta = e^{\theta}(\operatorname{sen}\Theta + \cos\Theta).$$

$$\int e^{\theta}\cos\Theta . d\Theta = \frac{e^{\theta}(\operatorname{sen}\Theta + \cos\Theta)}{2} + c.$$

20. $\displaystyle\int \frac{\ln x\, dx}{(x+1)} = \frac{x}{x+1}\ln x - \ln(x+1) + c.$

$$u = \ln x \qquad \int dv = \int \frac{1}{(x+1)^2}\, dx$$

$$du = \frac{1}{x}\, dx \qquad v = \frac{(x+1)^{-2+1}}{-2+1} = \frac{(x+1)^{-1}}{-1} = -\frac{1}{(x+1)} + c.$$

$$-\frac{\ln x}{x+1} - \int\left(-\frac{1}{x+1}\right)\left(\frac{1}{x}\right)dx = -\frac{\ln x}{x+1} + \int \frac{dx}{x(x+1)} = -\frac{\ln x}{x+1} + \int \frac{dx}{x^2 + x}$$

Solucionando: $x^2 + x$, completando con cuadrados.

$$x^2 + x + \frac{1}{4} - \frac{1}{4} = \left(x + \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2.$$

$$-\frac{\ln x}{x+1} + \int \frac{dx}{\left[\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]} = -\frac{\ln x}{x+1} + \frac{1}{2\left(\frac{1}{2}\right)}\ln\left|\frac{x + \frac{1}{2} - \frac{1}{2}}{x + \frac{1}{2} + \frac{1}{2}}\right| =$$

$$-\frac{\ln x}{x+1} + \ln\left|\frac{x}{x+1}\right| = -\frac{\ln x}{x+1} + \ln x - \ln(x+1) =$$

$$\frac{-\ln x+(\ln x)(x+1)}{x+1}-\ln(x+1)=\frac{(\ln x)(x+1-1)}{x+1}-\ln(x+1)=$$

$$\mathbf{\frac{x}{x+1}.\ln x-\ln(x+1)+c.}$$

21. $\int x^2 \text{ arc sen } x\, dx=\frac{x^3}{3}\text{arc sen } x+\frac{x^2+2}{9}\sqrt{1-x^2}+c.$

$$u=\text{arc sen } x \qquad \int dv=\int x^2 dx$$

$$du=\frac{1}{\sqrt{1-x^2}} \qquad v=\frac{x^3}{3}$$

$$\frac{x^3}{3}\text{arc sen } x-\int\left[\frac{x^3}{3}.\frac{1}{\sqrt{1-x^2}}\right]=\frac{x^3}{3}\text{arc sen } x-\frac{1}{3}\int \mathbf{(1-x^2)}^{-\frac{1}{2}}x^3dx$$

$$\frac{x^3}{3}\text{arc sen } x-\frac{1}{3}\int x^2.(1-x^2)^{-\frac{1}{2}}\, x\, dx$$ **.Integrando por partes la 2**$^{\mathbf{da}}$ **integral**

$$u=x^2 \qquad \int dv=\int(1-x^2)^{-\frac{1}{2}}\, x\, dx \int dv=\left(-\frac{1}{2}\right)\int(1-x^2)^{-\frac{1}{2}}.(-2)x\, dx$$

$$du=2x\, dx \qquad v=-\frac{1}{2}\left[\frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]=-\frac{(1-x^2)^{\frac{1}{2}}}{2\left(\frac{1}{2}\right)}=-(1-x^2)^{\frac{1}{2}}$$

$$v=-\sqrt{1-x^2}$$

$$\frac{x^3}{3}\text{arc sen } x-\frac{1}{3}\left[(x^2)\left(-\sqrt{1-x^2}\right)-\int\left(-\sqrt{1-x^2}\right)(2x)\, dx\right]$$

$$\frac{x^3}{3}\text{arc sen } x-\frac{1}{3}\left[(x^2)\left(-\sqrt{1-x^2}\right)-\int(1-x^2)^{\frac{1}{2}}.(-2x)\, dx\right]$$

$$\frac{x^3}{3}\text{arc sen } x+\frac{x^2\sqrt{1-x^2}}{3}+\frac{1}{3}\left[\frac{(1-x^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]$$

$$\frac{x^3}{3}\,\text{arc sen } x + \frac{x^2\sqrt{1-x^2}}{3} + \frac{1}{3}\left[\frac{(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$\frac{x^3}{3}\,\text{arc sen } x + \frac{x^2\sqrt{1-x^2}}{3} + \frac{2}{9}(1-x^2)\sqrt{1-x^2}$$

$$\frac{x^3}{3}\,\text{arc sen } x + \sqrt{1-x^2}\left[\frac{x^2}{3} + \frac{2(1-x^2)}{9}\right]$$

$$\frac{x^3}{3}\,\text{arc sen } x + \sqrt{1-x^2}\left[\frac{3x^2+2-2x^2}{9}\right]$$

$$\frac{x^3}{3}\,\text{arc sen } x + \sqrt{1-x^2}\left[\frac{x^2+2}{9}\right]$$

$$\frac{x^3}{3}\,\text{arc sen } x + \frac{x^2+2}{9}.\sqrt{1-x^2} + c\,.$$

22. $$\int \frac{\ln(x+1)\,dx}{\sqrt{x+1}} = 2\sqrt{x+1}\left[\ln(x+1)-2\right] + c.$$

$$u = \ln(x+1) \qquad\qquad dv = \int \frac{1}{\sqrt{x+1}} = \int (x+1)^{\frac{-1}{2}}\,dx$$

$$du = \frac{1}{x+1}\,dx \qquad\qquad v = \frac{(x+1)^{-\frac{1}{2}+1}}{\frac{-1}{2}+1} = \frac{(x+1)^{\frac{1}{2}}}{\frac{1}{2}} = 2(x+1)^{\frac{1}{2}}$$

$$2(x+1)^{\frac{1}{2}}.\ln(x+1) - \int 2(x+1)^{\frac{1}{2}}\,.\frac{1}{(x+1)^{\frac{2}{2}}}\,dx$$

$$2(x+1)^{\frac{1}{2}}\ln(x+1) - 2\int \frac{dx}{(x+1)^{\frac{1}{2}}} = 2(x+1)^{\frac{1}{2}}\ln(x+1) - 2\int (x+1)^{-\frac{1}{2}}\,dx$$

$$2(x+1)^{\frac{1}{2}}\left[\ln(x+1)-2\right] + c\,.$$

23. $\int \frac{x\,e^{x}\,dx}{(1+x)^2} = \frac{e^{x}}{\mathbf{1+x}} + c.$

$$\int e^{x}.x.\frac{1}{(1+x)^2}.dx$$

$u = e^{x}.x \qquad\qquad \int dv = \int \frac{1}{(1+x)^2}\,dx$

$du = e^{x} + xe^{x} \qquad\qquad v = \int (1+x)^{-2}\,dx$

$du = e^{x}(1+x) = e^{x}(x+1) \qquad v = \frac{(1+x)^{-2+1}}{-2+1} = \frac{(1+x)^{-1}}{-1} = -\frac{1}{(1+x)}.$

$$(e^{x}.x)\left(-\frac{1}{1+x}\right) - \int\left(-\frac{1}{1+x}\right).(e^{x})(x+1)dx$$

$$\left(\frac{-xe^{x}}{1+x}\right) + \int e^{x}dx = \frac{-xe^{x}}{1+x} + e^{x} = \frac{-xe^{x} + e^{x}(1+x)}{1+x} = \frac{e^{x}(-x+1+x)}{1+x} =$$

$$\frac{e^{x}(1)}{1+x} = \frac{e^{x}}{1+x} + c.$$

24. $\int e^{-t}\cos \pi t\,.\,dt = \frac{e^{-t}(\pi \operatorname{sen} \pi t - \cos \pi t)}{\pi^2 + 1}$

$u = e^{-t} \quad ; \quad dv = \cos \pi t\,.\,dt$

$du = -e^{-t} \;;\; \int dv = \int \cos \pi t\,.\,dt = \left(\frac{1}{\pi}\right)\int \cos \pi t.(\pi)\,dt$

$du = -e^{-t} \qquad\qquad v = \frac{\operatorname{sen} \pi t}{\pi}.$

$$\int e^{-t}\cos \pi t.\,dt = (e^{-t})\left(\frac{\operatorname{sen} \pi t}{\pi}\right) - \int\left(\frac{\operatorname{sen} \pi t}{\pi}\right)(-e^{-t})\,dt$$

$$\int e^{-t}\cos \pi t.\,dt = (e^{-t})\left(\frac{\operatorname{sen} \pi t}{\pi}\right) + \frac{1}{\pi}\int e^{-t}\,.\,\operatorname{sen} \pi t\,dt$$

$u = e^{-t} \quad ; \quad dv = \operatorname{sen} \pi t \;;\; \int dv = \int \operatorname{sen} \pi t\,dt.$

$du = -e^{-t}\,; \qquad v = \int \operatorname{sen} \pi t\,dt = (1/\pi)\int \operatorname{sen} \pi t.(\pi)\,dt = -\frac{\cos \pi t}{\pi}$

$$\int e^{-t}\cos \pi t\,.\,dt = \left(e^{-t}\right)\left(\frac{\text{sen } \pi t}{\pi}\right) + \frac{1}{\pi}\int e^{-t}.\ \text{sen } \pi t\, dt$$

$$\int e^{-t}\cos \pi t\,.\,dt = \left(e^{-t}\right)\left(\frac{\text{sen } \pi t}{\pi}\right) + \frac{1}{\pi}\left[\left(e^{-t}\right)\left(\frac{-\cos \pi t}{\pi}\right)\right] - \int\left(\frac{-\cos \pi t}{\pi}\right)\left(e^{-t}\right)dt$$

$$\int e^{-t}\cos \pi t\,.\,dt = \left(e^{-t}\right)\left(\frac{\text{sen } \pi t}{\pi}\right) - \frac{e^{-t}.\ \cos \pi t}{\pi^2} - \frac{1}{\pi^2}\int\left(e^{-t}\right)\left(\cos \pi t\right)dt$$

$$\int (e^{-t})(\cos \pi t)\, dt + \frac{1}{\pi^2}\int\left(e^{-t}\right)\left(\cos \pi t\right)dt = \left(e^{-t}\right)\left(\frac{\text{sen } \pi t}{\pi}\right) - \frac{e^{-t}.\ \cos \pi t}{\pi^2}$$

$$\left[\int (e^{-t})(\cos \pi t)\, dt\right]\left[1 + \frac{1}{\pi^2}\right] = \left(e^{-t}\right)\left(\frac{\text{sen } \pi t}{\pi}\right) - \frac{e^{-t}.\ \cos \pi t}{\pi^2}$$

$$\left[\int (e^{-t})(\cos \pi t)\, dt\right] = \frac{\left(e^{-t}\right)\left(\frac{\text{sen } \pi t}{\pi}\right) - \frac{e^{-t}.\ \cos \pi t}{\pi^2}}{\left[1 + \frac{1}{\pi^2}\right]}$$

$$\int (e^{-t})(\cos \pi t)\, dt = \frac{\frac{e^{-t}\ \text{sen } \pi t}{\pi} - \frac{e^{-t}\ \cos \pi t}{\pi^2}}{\frac{\pi^2 + 1}{\pi^2}}$$

$$\int (e^{-t})(\cos \pi t)\, dt = \frac{e^{-t}(\pi \text{ sen } \pi t - \cos \pi t)}{\pi^2 + 1} + c.$$

26. $\int x.\cos^2 2x\ dx$

$u = x \qquad dv = \cos^2 2x\ dx \ ; \ \int dv = \int \cos^2 2x\ dx$

$du = dx \qquad v = \int\left(\frac{1}{2} + \frac{1}{2}\cos 4x\right)dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 4x\ dx$

$$v = \left(\frac{1}{2}x\right) + \left(\frac{1}{2}.\frac{1}{4}\right)\int(\cos 4x).(4)\ dx$$

$$v = \frac{1}{2}x + \frac{1}{8}\left[\text{sen } 4x\right] = \frac{x}{2} + \frac{\text{sen } 4x}{8}$$

$$\int x.\cos^2 2x\ dx = x\left[\frac{x}{2} + \frac{\text{sen } 4x}{8}\right] - \int\left[\frac{x}{2} + \frac{\text{sen } 4x}{8}\right]dx$$

$$= \frac{x^2}{2} + \frac{x \text{ sen } 4x}{8} - \frac{1}{2}\int x\ dx - \frac{1}{8}\int \text{sen } 4x.\ dx =$$

$$= \frac{x^2}{2} + \frac{x \text{ sen } 4x}{8} - \frac{1}{2}.\frac{x^2}{2} - \frac{1}{8}.\frac{1}{4}\int \text{sen } 4x\ .(4)dx$$

$$= \frac{x^2}{2} + \frac{x \text{ sen } 4x}{8} - \frac{x^2}{4} - \frac{1}{32}\left[-\cos 4x\right] =$$

$$= \frac{x^2}{2} - \frac{x^2}{4} + \frac{x \text{ sen } 4x}{8} + \frac{\cos 4x}{32} =$$

$$= \frac{2x^2}{4} - \frac{x^2}{4} + \frac{x \text{ sen } 4x}{8} + \frac{\cos 4x}{32} = \frac{x^2}{4} + \frac{x \text{ sen } 4x}{8} + \frac{\cos 4x}{32} + c\ .$$

25. $\int x.\sec^2 \frac{x}{2}\ dx.$

$u = x \qquad dv = \sec^2 \frac{x}{2}\ dx \ ; \ \int dv = 2\int \sec^2 \frac{x}{2}.(1/2)\ dx.$

$du = dx \qquad v = 2\ \text{tg} \frac{x}{2}$

$$(x)\left(2\ \text{tg} \frac{x}{2}\right) - 2\int\left(\text{tg} \frac{x}{2}\right)dx = 2x\ \text{tg} \frac{x}{2} - 2(2)\int\left(\text{tg} \frac{x}{2}\right)(1/2)\ dx =$$

$$2x\ \text{tg} \frac{x}{2} - 4 \ln \sec \frac{x}{2} + c\ .$$

27. $\int x^2 \cos x\ dx$

$u = x^2 \qquad dv = \cos x\ dx \ ; \ \int dv = \int \cos x\ dx$

$du = 2x\ dx \qquad v = \text{sen } x$

$$= x^2 \text{ sen } x - \int(\text{sen } x)(\ 2x\ )\ dx\ = x^2 \text{ sen } x - 2\int x\ .\text{sen } x\ \ dx =$$

integrando por partes la integral : $\int x\ .\text{sen } x\ \ dx =$

$u = x \ ; \ du = dx \ ; \ dv = \text{sen } x\ dx \ ; \ \int dv = \int \text{sen } x\ dx \ ; \ v = -\cos x.$

$$\int x^2 \cos x\ dx = x^2 \text{ sen } x - 2\int x\ .\text{sen } x\ \ dx =$$

$$\int x^2 \cos x\ dx = x^2 \text{ sen } x - 2\left[(x)(-\cos x) - \int(-\cos x)\ dx\right]$$

$$\int x^2 \cos x\ dx = x^2 \text{ sen } x + 2x \cos x - 2\int \cos x\ dx$$

$$\int x^2 \cos x\, dx = x^2 \operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x + c\,.$$

28. $\int \operatorname{arc\,sen} mx\, dx.$

$u = \operatorname{arc\,sen} mx \qquad dv = dx$

$du = \dfrac{m}{\sqrt{1-m^2x^2}} \qquad v = x$

$$x \operatorname{arc\,sen} mx - \int \frac{mx}{\sqrt{1-m^2x^2}}\, dx$$

$$x \operatorname{arc\,sen} mx - \left(-\frac{1}{2m}\right)\int \left(1-m^2x^2\right)^{-1/2}.(-2m)\, mx.dx$$

$$x \operatorname{arc\,sen} mx + \left(\frac{1}{2m}\right)\left[\frac{\left(1-m^2x^2\right)^{1/2}}{1/2}\right]$$

$$x \operatorname{arc\,sen} mx + \frac{2\sqrt{1-m^2x^2}}{2m} = x \operatorname{arc\,sen} mx + \frac{\sqrt{1-m^2x^2}}{m} + c\,.$$

29. $\int \operatorname{arc\,cot} \frac{x}{2}\, dx$

$u = \operatorname{arc\,cot} \frac{x}{2} \qquad dv = dx\,.$

$du = \dfrac{-\frac{1}{2}}{1+\frac{x^2}{4}} = \dfrac{-2}{x^2+4} \qquad v = x\,.$

$$x \operatorname{arc\,cot} \frac{x}{2} - \int \frac{-2x}{x^2+4}.dx = x \operatorname{arc\,cot} \frac{x}{2} + \int \frac{(2x)}{x^2+4}\, dx = x \operatorname{arc\,cot} \frac{x}{2} + \ln$$
$$x \operatorname{arc\,cot} \frac{x}{2} + \ln\left(x^2+4\right) + c\,.$$

30. $\int \operatorname{arc\,cos} \frac{1}{x}\, dx\,.$

$u = \operatorname{arc\,cos} \frac{1}{x}\,. \qquad dv = dx \;;\; v = x\,.$

$$du = \frac{-\frac{1}{x^2}}{\sqrt{1-\left(\frac{1}{x}\right)^2}} = \frac{-\frac{1}{x^2}}{\sqrt{1-\frac{1}{x^2}}} = \frac{-\frac{1}{x^2}}{\sqrt{\frac{x^2-1}{x^2}}} = \frac{-\frac{1}{x^2}}{\frac{\sqrt{x^2-1}}{\sqrt{x^2}}} = \frac{-\frac{1}{x^2}}{\frac{\sqrt{x^2-1}}{x}} = -\frac{1}{x\sqrt{x^2-1}}$$

$$x\ \text{arc cos}\ \frac{1}{x} - \int (x)\left[-\frac{1}{x\sqrt{x^2-1}}\right]dx = x\ \text{arc cos}\ \frac{1}{x} + \int\left[\frac{1}{\sqrt{x^2-1}}\right]dx$$

$$x\ \text{arc cos}\ \frac{1}{x} + \ln\left[x+\sqrt{x^2-1}\right] + c\,.$$

31. $\int \text{arc sec}\ \frac{1}{y}\ dy$

$u = \text{arc sec}\ \frac{1}{y}\,.$ $\qquad dv = dy \quad ; \quad v = y$

$$du = \frac{-\frac{1}{y^2}}{\frac{1}{y}\sqrt{\frac{1}{y^2}-1}} = \frac{-\frac{1}{y}}{\sqrt{\frac{1-y^2}{y^2}}} = \frac{-\frac{1}{y}}{\frac{\sqrt{1-y^2}}{\sqrt{y^2}}} = \frac{-\frac{1}{y}}{\frac{\sqrt{1-y^2}}{y}} = -\frac{1}{\sqrt{1-y^2}}$$

$$y\ \text{arc sec}\ \frac{1}{y} - \int \frac{-y}{\sqrt{1-y^2}}\,.\,dy = y\ \text{arc sec}\ \frac{1}{y} - \frac{1}{2}\int\left[\left(1-y^2\right)^{-1/2}.(-2)y\right]dy =$$

$$y\ \text{arc sec}\ \frac{1}{y} - \frac{1}{2}\left[\frac{\left(1-y^2\right)^{-1/2+1}}{-1/2+1}\right] = y\ \text{arc sec}\ \frac{1}{y} - \frac{1}{2}\left[\frac{\left(1-y^2\right)^{1/2}}{1/2}\right]$$

$$y\ \text{arc sec}\ \frac{1}{y} - \frac{2}{2}\left[\left(1-y^2\right)^{1/2}\right] = y\ \text{arc sec}\ \frac{1}{y} - \sqrt{1-y^2} + c\,.$$

32. $\int \text{arc csc nt dt}$

$u = \text{arc csc nt}$ $\qquad dv = dt\,.$

$du = \frac{-n}{nt\sqrt{n^2t^2-1}}\,.\,dt$ $\qquad v = t\,.$

$$t\ \text{arc csc nt} - \int (t)\left[\frac{-n}{nt\sqrt{n^2t^2-1}}\right].\ dt = t\ \text{arc csc nt} + \int \frac{dt}{\sqrt{(nt)^2-1^2}}$$

$$t\ \text{arc csc nt} + \ln\left[nt\sqrt{n^2t^2-1}\right] + c.$$

33. $\int \text{arc sen}\ \sqrt{\frac{x}{2}}\ dx$

$u = \text{arc sen}\ \sqrt{\frac{x}{2}}$ $\qquad dv = dx$

$du = \frac{1}{2\sqrt{x}\sqrt{2-x}}$ $\qquad v = x$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \int (x)\left[\frac{1}{2\sqrt{x}\sqrt{2-x}}\right]dx =$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \frac{1}{2}\int\left[\frac{x}{\sqrt{x(2-x)}}\right]dx =$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \frac{1}{2}\int\left[\frac{x}{\sqrt{2x-x^2}}\right]dx =$$

$$u = x \qquad \int dv = \int \frac{1}{\sqrt{2x-x^2}} = \int \frac{1}{\sqrt{1^2-(x-1)^2}} =$$

$$du = dx \qquad v = \text{arc sen } (x-1)$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \frac{1}{2}\left[x \ \text{arc sen } (x-1) - \int \text{arc sen } (x-1)\ dx\right]$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \frac{x \ \text{arc sen } (x-1)}{2} + \frac{1}{2}\left[(x-1)\,\text{arcsen}(x-1) + \sqrt{1-(x-1)^2}\right]$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \frac{x \ \text{arc sen } (x-1)}{2} + \frac{(x-1)\,\text{arcsen}(x-1)}{2} + \frac{\sqrt{1-(x-1)^2}}{2}$$

Factorizando : $\frac{\text{arcsen}(x-1)}{2}$.

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \left[\frac{\text{arcsen}(x-1)}{2}\right]\left[x-(x-1)\right] + \frac{\sqrt{1-(x-1)^2}}{2}$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \left[\frac{\text{arcsen}(x-1)}{2}\right]\left[x-x+1\right] + \frac{\sqrt{1-(x-1)^2}}{2}$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \left[\frac{\text{arcsen}(x-1)}{2}\right]\left[1\right] + \frac{\sqrt{1-(x-1)^2}}{2}$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \frac{\text{arcsen}(x-1)}{2} + \frac{\sqrt{1-(x-1)^2}}{2}$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \left[\frac{1}{2}\right]\left[\text{arcsen}(x-1) - \sqrt{1-(x-1)^2}\right]$$

$$x \ \text{arc sen} \sqrt{\frac{x}{2}} - \frac{\text{arcsen}(x-1) - \sqrt{1-(x-1)^2}}{2} + c\,.$$

34. $\int x^3 \text{ arc sen} x \; dx$.

$u = x^3 \qquad\qquad dv = \text{arc sen} x$

$du = 3x^2\,dx \qquad v = x \text{ arc sen} x + \int \sqrt{1-x^2}$

$$\int x^3 \text{arc sen} x \; dx = x^3\left[x \text{ arc sen} x + \sqrt{1-x^2} - \int (3x^2)\left(x \text{ arc sen} x + \sqrt{1-x^2}\right)\right]$$

$$\int x^3 \text{arc sen} x \; dx = x^4 \text{arc sen} x + x^3\sqrt{1-x^2} - 3\int x^3 \text{ arcsen } x\,dx - 3\int x^2\sqrt{1-x^2}\,dx$$

$$\int x^3 \text{arc sen} x \; dx = x^4 \text{arc sen} x + x^3\sqrt{1-x^2} - 3\int x^3 \text{ arcsen } x\,dx - 3\int x^2 (1-x^2)^{1/2}.dx$$

$$4\int x^3 \text{arc sen} x \; dx = x^4 \text{arc sen} x + x^3\sqrt{1-x^2} - 3\left(-\frac{1}{2}\right)\int x\,.(1-x^2)^{1/2}\,(-2)x \; dx.$$

$$4\int x^3 \text{arc sen} x \; dx = x^4 \text{arc sen} x + x^3\sqrt{1-x^2} + \frac{3}{2}\int x\,.(1-x^2)^{1/2}\,(-2)x \; dx.$$

$u = x \quad ; \quad \int dv = \int (1-x^2)^{1/2}.(-2)x \; dx$.

$du = dx \; ; \quad v = \dfrac{\left(1-x^2\right)^{1/2+1}}{1/2+1}$

$v = \dfrac{2\left(1-x^2\right)^{3/2}}{3}$

$$4\int x^3 \text{arc sen} x \; dx = x^4 \text{arc sen} x + x^3\sqrt{1-x^2} + \frac{3}{2}\left[\frac{2x(1-x^2)^{3/2}}{3} - \right]$$